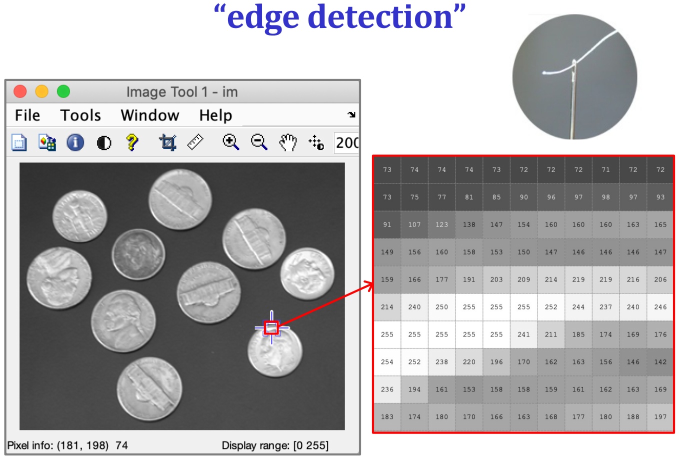
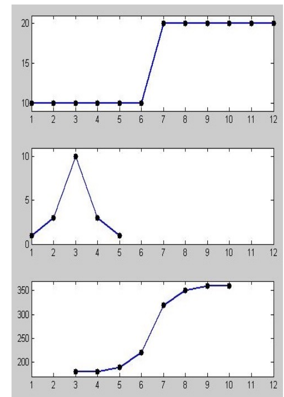
Detecting intensity changes in an image
"edge detection"



1

Convolution in one dimension



1	31	16	30	18	30	18	30	13	30	13	1
10	10	10	10	10	10	20	20	20	20	20	20

intensity
I(x)

convolution operator
G(x)

1	3	10	3	1

convolution result

0	0	180	180	190	220	320	350	360	360	0	0

G(x) * I(x)

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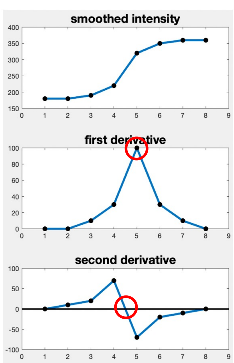
Computing derivatives with convolution



-1	+1

smoothed intensity

180	180	190	220	320	350	360	360

-1	+1

1st derivative

0	0	10	30	100	30	10	0

2nd derivative

0	0	10	20	70	-70	-20	-10

Why is this operator a 2nd derivative?

+1	-2	+1

I(x)

V_1	V_2	V_3

I'(x)

	V_2-V_1	V_3-V_2

I''(x)

		$(V_3-V_2) - (V_2-V_1)$

$= V_3 - 2V_2 + V_1$

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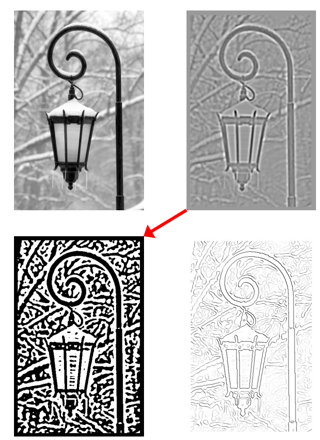
Detecting intensity changes in a 2D image

image

image after smoothing and second derivative

(convolution with Laplacian-of-Gaussian operator)



black = negative

white = positive

zero-crossing contours

"edge detection"

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Computing Laplacian with convolution

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	10	10	0	0	0	0
0	0	0	10	10	10	10	0	0	0
0	0	10	10	10	10	10	10	0	0
0	0	10	10	10	10	10	10	0	0
0	0	0	10	10	10	10	0	0	0
0	0	0	0	10	10	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Convolution operator

0	1	0
1	-4	1
0	1	0

Laplacian

+1	-2	+1

+

	+1	
	-2	
	+1	

$$\nabla^2 = \left(\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \right)$$

convolution result

0	0	0	0	0	0	0	0	0	0
0	0	0	0	10	10	0	0	0	0
0	0	0	20	-20	-20	20	0	0	0
0	0	20	-20	0	0	-20	20	0	0
0	10	-20	0	0	0	0	-20	10	0
0	10	-20	0	0	0	0	-20	10	0
0	0	20	-20	0	0	-20	20	0	0
0	0	0	20	-20	-20	20	0	0	0
0	0	0	0	10	10	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(with zero-crossings highlighted)

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What does this operator do?

1	1	1	1	1
1	1	1	1	1
10	10	10	1	1
10	10	10	1	1
10	10	10	1	1

image

Convolution operator

1	1	1
-2	-2	-2
1	1	1

	27	18		
	-27	-18		
		0	0	

convolution result

smoothing?

1st or 2nd derivative?

directional or non-directional derivative?

can we locate intensity changes from result?

can we detect intensity changes at all orientations?

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images

1	1	3	7	10	10
1	1	3	7	10	10
1	1	3	7	10	10
1	1	3	7	10	10
1	1	3	7	10	10
1	1	3	7	10	10

1	1	1	1	1	1
1	1	1	1	1	1
3	3	3	3	3	3
7	7	7	7	7	7
10	10	10	10	10	10
10	10	10	10	10	10

convolution operator

-1	-1	-1
0	0	0
+1	+1	+1

smoothing?

1st or 2nd derivative?

directional or not?

locate intensity changes?

all orientations?

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

convolution results

0	0	0	0	0	0
0	6	6	6	6	0
0	18	18	18	18	0
0	21	21	21	21	0
0	9	9	9	9	0
0	0	0	0	0	0

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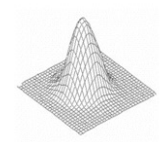
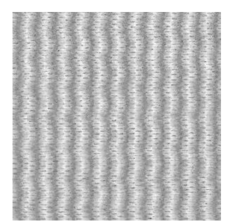
Smoothing a 2D image

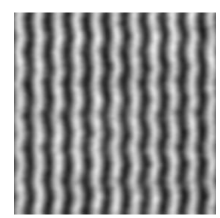
To smooth a 2D image I(x,y), we convolve with a 2D Gaussian:

$$G(x,y) = \left(\frac{1}{\sigma^2}\right) e^{\left(\frac{-(x^2+y^2)}{2\sigma^2}\right)}$$





image



result of convolution

G(x,y) * I(x,y)

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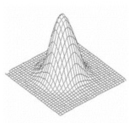
Constructing the convolution operator

To smooth 2D image I(x,y), convolve with 2D Gaussian

$$G(x,y) = \left(\frac{1}{\sigma^2}\right) e^{\left(\frac{-(x^2+y^2)}{2\sigma^2}\right)}$$



$\sigma = 2$

(values scaled by 100)

y ↑ , x →

0.05	0.15	0.36	0.67	0.97	1.10	0.97	0.67	0.36	0.15	0.05
0.15	0.46	1.10	2.05	2.99	3.38	2.99	2.05	1.10	0.46	0.15
0.36	1.10	2.63	4.92	7.16	8.12	7.16	4.92	2.63	1.10	0.36
0.67	2.05	4.92	9.20	13.38	15.16	13.38	9.20	4.92	2.05	0.67
0.97	2.99	7.16	13.38	19.47	22.06	19.47	13.38	7.16	2.99	0.97
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.97	2.99	7.16	13.38	19.47	22.06	19.47	13.38	7.16	2.99	0.97
0.67	2.05	4.92	9.20	13.38	15.16	13.38	9.20	4.92	2.05	0.67
0.36	1.10	2.63	4.92	7.16	8.12	7.16	4.92	2.63	1.10	0.36
0.15	0.46	1.10	2.05	2.99	3.38	2.99	2.05	1.10	0.46	0.15
0.05	0.15	0.36	0.67	0.97	1.10	0.97	0.67	0.36	0.15	0.05

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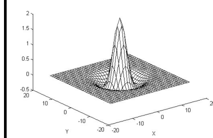
Sampling the Laplacian-of-Gaussian

$$\nabla^2 G = \frac{1}{\sigma^2}\left(\frac{r^2}{\sigma^2} - 2\right) e^{-\frac{r^2}{2\sigma^2}}$$

$r^2 = x^2 + y^2 \qquad w = 2\sqrt{2}\sigma$



(displayed with sign reversed)

w = 6

(values scaled by 10)

y ↑ , x →, w

-0.05	-0.13	-0.30	-0.54	-0.81	-1.02	-1.10	-1.02	-0.81	-0.54	-0.30	-0.13	-0.05
-0.13	-0.35	-0.75	-1.27	-1.77	-2.10	-2.21	-2.10	-1.77	-1.27	-0.75	-0.35	-0.13
-0.30	-0.75	-1.46	-2.21	-2.65	-2.69	-2.63	-2.69	-2.65	-2.21	-1.46	-0.75	-0.30
-0.54	-1.27	-2.21	-2.71	-2.10	-0.73	[illegible]	-0.73	-2.10	-2.71	-2.21	-1.27	-0.54
-0.81	-1.77	-2.65	-2.10	0.91	5.10	7.12	5.10	0.91	-2.10	-2.65	-1.77	-0.81
-1.02	-2.10	-2.69	-0.73	5.10	12.46	15.91	12.46	5.10	-0.73	-2.69	-2.10	-1.02
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.02	-2.10	-2.69	-0.73	5.10	12.46	15.91	12.46	5.10	-0.73	-2.69	-2.10	-1.02
-0.81	-1.77	-2.65	-2.10	0.91	5.10	7.12	5.10	0.91	-2.10	-2.65	-1.77	-0.81
-0.54	-1.27	-2.21	-2.71	-2.10	-0.73	-0.00	-0.73	-2.10	-2.71	-2.21	-1.27	-0.54
-0.30	-0.75	-1.46	-2.21	-2.65	-2.69	-2.63	-2.69	-2.65	-2.21	-1.46	-0.75	-0.30
-0.13	-0.35	-0.75	-1.27	-1.77	-2.10	-2.21	-2.10	-1.77	-1.27	-0.75	-0.35	-0.13
-0.05	-0.13	-0.30	-0.54	-0.81	-1.02	-1.10	-1.02	-0.81	-0.54	-0.30	-0.13	-0.05

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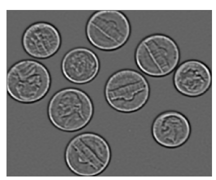
Computing the contrast of intensity changes

$[\nabla^2 G(x,y)] * I(x,y)$



zero-crossings with slope displayed as darkness

-18	-12	13
-10	14	17
12	20	26

$$slope = \sqrt{dx^2 + dy^2}$$

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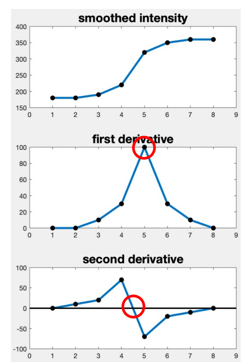
Computing derivatives with convolution



smoothed intensity

-1	+1

180	180	190	220	320	350	360	360

1st derivative

-1	+1

0	0	10	30	100	30	10	0

2nd derivative

0	0	10	20	70	-70	-20	-10

Why is this operator a 2nd derivative?

+1	-2	+1

I(x)

V_1	V_2	V_3

I′(x)

	V_2-V_1	V_3-V_2

I′′(x)

		$(V_3-V_2) - (V_2-V_1)$

$= V_3 - 2V_2 + V_1$

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