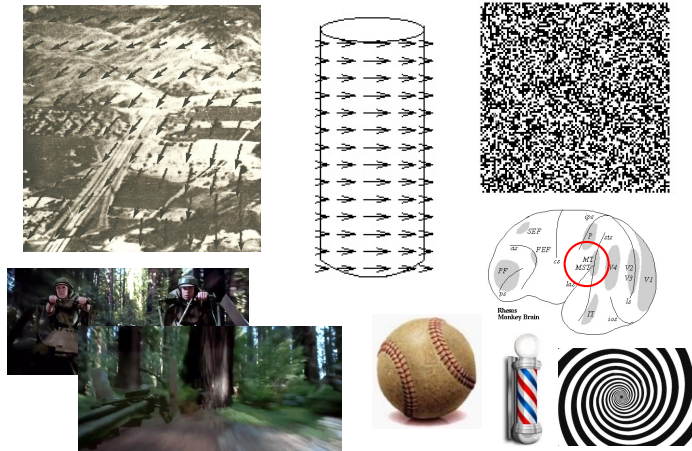
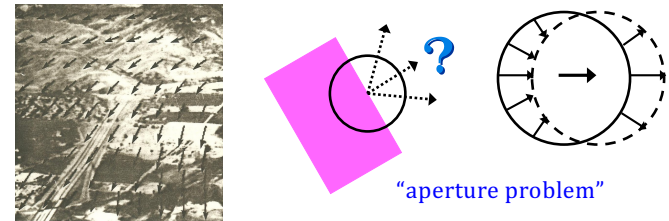


Analysis of visual motion



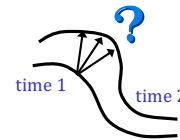
1

Measuring image motion



velocity field

"local" motion detectors only measure *component of motion perpendicular to moving edge*

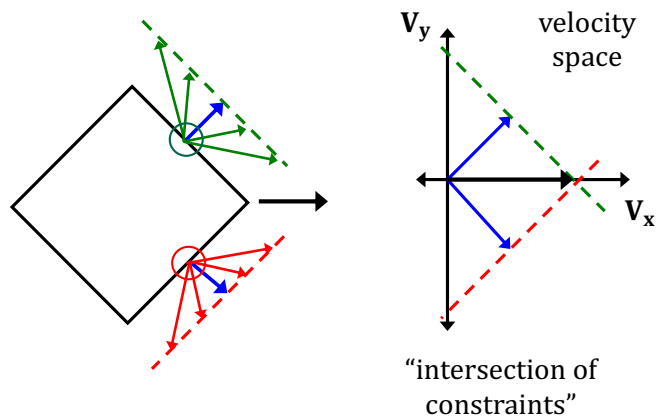


2D velocity field not determined *uniquely* from the changing image

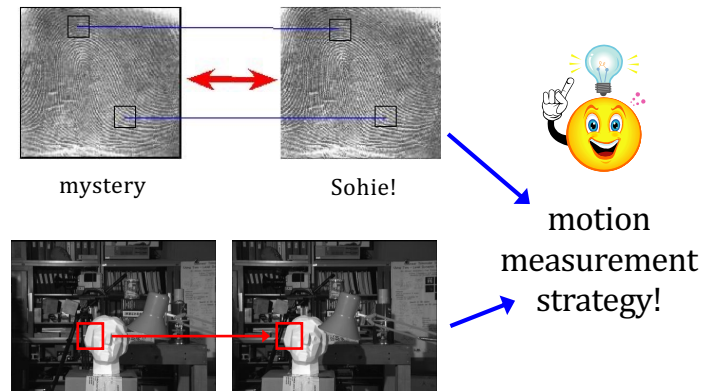
need *additional constraint* to compute a unique velocity field

2

Option 1: Assume *pure translation*



3



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Practical considerations for methods based on pure translation:

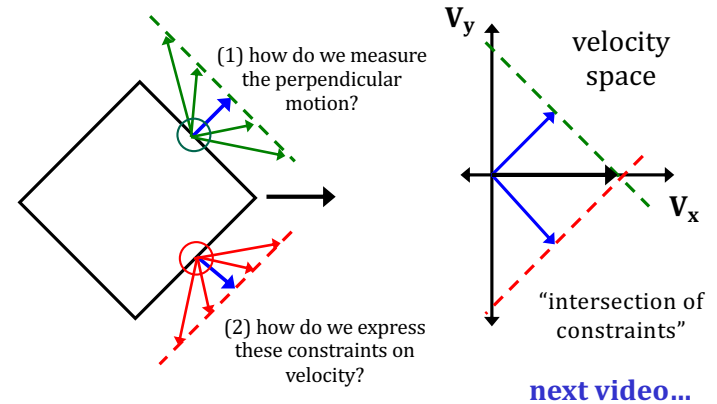
- Error in initial motion measurements
- Local image features may have small range of orientations
- Velocities not constant locally

But... such strategies are good for

- detecting sudden movements
- tracking
- detecting boundaries

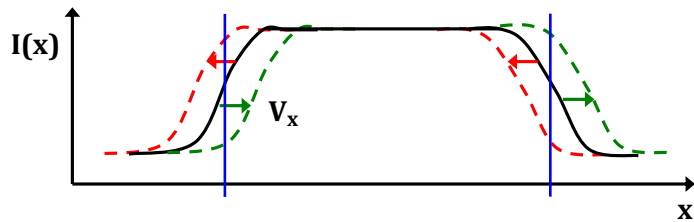
5

Goals for the rest of this video



6

Measuring motion in one dimension



V_x = velocity in x direction

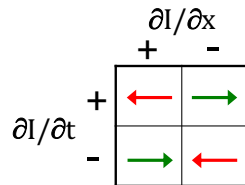
• rightward movement: $V_x > 0$

• leftward movement: $V_x < 0$

• speed: $|V_x|$

• pixels/time step

$$V_x = - \frac{\partial I / \partial t}{\partial I / \partial x}$$



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Measuring motion components in 2-D

(1) gradient of image intensity

$$\nabla I = (\partial I / \partial x, \partial I / \partial y)$$

(2) time derivative

$$\partial I / \partial t$$

(3) velocity along gradient:

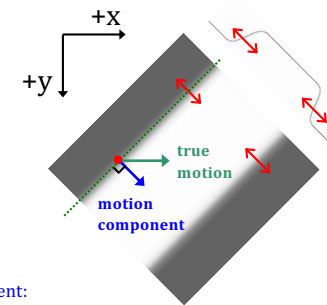
$$v^\perp$$

• movement in direction of gradient:

$$v^\perp > 0$$

• movement opposite direction of gradient:

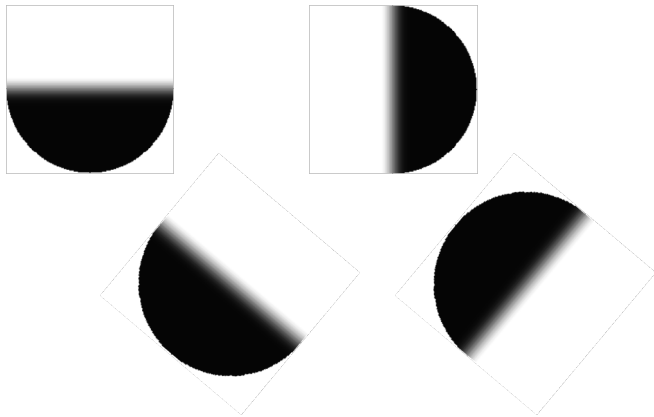
$$v^\perp < 0$$



$$v^\perp = - \frac{\partial I / \partial t}{|\nabla I|} = - \frac{\partial I / \partial t}{[(\partial I / \partial x)^2 + (\partial I / \partial y)^2]^{1/2}}$$

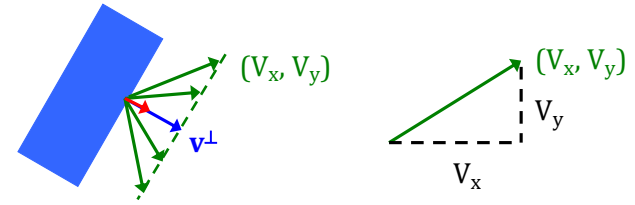
8

Direction of the gradient
 $\nabla I = (\partial I / \partial x, \partial I / \partial y)$



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2-D velocities (V_x, V_y) consistent with v^\perp



All (V_x, V_y) such that the component of (V_x, V_y) in the direction of the gradient is v^\perp

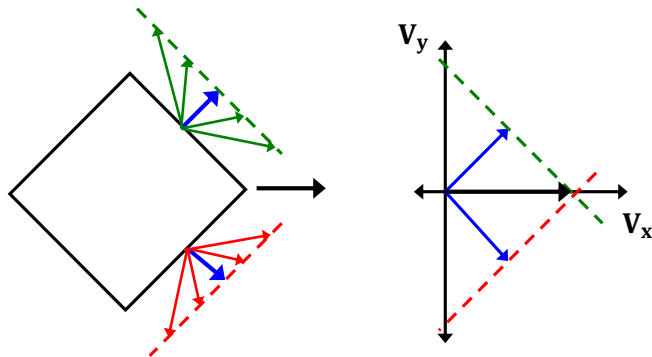
(u_x, u_y) : unit vector in direction of gradient

Use the dot product: $(V_x, V_y) \cdot (u_x, u_y) = v^\perp$

$$V_x u_x + V_y u_y = v^\perp$$

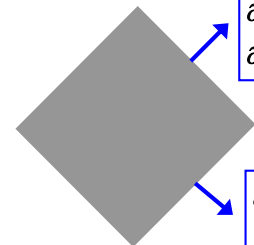
10

Time-out exercise



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Details...



$$\begin{aligned} \partial I / \partial x &= 10 \\ \partial I / \partial y &= -10 \\ \partial I / \partial t &= -30 \end{aligned}$$

$$\begin{aligned} \partial I / \partial x &= 10 \\ \partial I / \partial y &= 10 \\ \partial I / \partial t &= -30 \end{aligned}$$

$(V_x, V_y) ??$

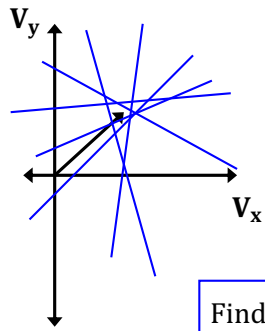


solve for V_x, V_y

- For each component:
- (1) u_x
 - (2) u_y
 - (3) v^\perp
 - (4) $V_x u_x + V_y u_y = v^\perp$

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In practice...



Previously...

$$V_x u_x + V_y u_y = v^\perp$$

New strategy:

Find (V_x, V_y) that **best fits** all motion components together

$$\text{Find } (V_x, V_y) \text{ that minimizes: } \sum (V_x u_x + V_y u_y - v^\perp)^2$$

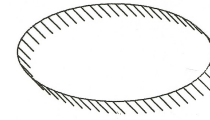
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Option 2: Smoothness assumption:

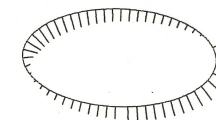
Compute a velocity field that:

- (1) is consistent with local measurements of image motion (perpendicular components)
- (2) has the *least amount of variation* possible

Pure Translation:



true & smoothest velocity field

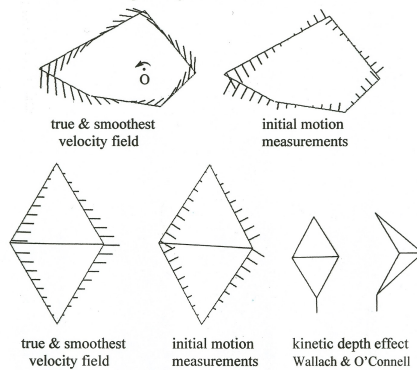


initial motion measurements

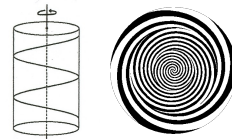
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When is the smoothest velocity field correct?

Rotation of rigid objects in 2D and 3D:



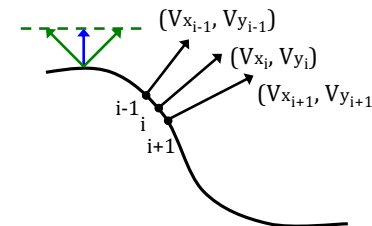
When is it wrong?



motion illusions

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Computing the smoothest velocity field



motion components:

$$V_{x_i} u_{x_i} + V_{y_i} u_{y_i} = v^\perp_i$$

change in velocity:

$$(V_{x_{i+1}} - V_{x_i}, V_{y_{i+1}} - V_{y_i})$$

Find (V_{x_i}, V_{y_i}) that minimize:

$$\sum (V_{x_i} u_{x_i} + V_{y_i} u_{y_i} - v^\perp_i)^2 + \lambda [(V_{x_{i+1}} - V_{x_i})^2 + (V_{y_{i+1}} - V_{y_i})^2]$$

deviation from image motion measurements + variation in velocity field

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