

Aperture Problem

velocity field

velocity space

“local” motion detectors only measure *component of motion* perpendicular to moving edge

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Computing 2D velocity from motion components

(1) how do we measure the perpendicular motion?

(2) how do we express these constraints on velocity?

velocity space

“intersection of constraints”

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Measuring motion in one dimension

$I(x)$

x

V_x

V_x = velocity in x direction

- rightward movement: $V_x > 0$
- leftward movement: $V_x < 0$
- speed: $|V_x|$
- pixels/time step

$$V_x = - \frac{\partial I / \partial t}{\partial I / \partial x}$$

	$\partial I / \partial x$	
+	←	→
-	→	←

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Measuring motion components in 2-D

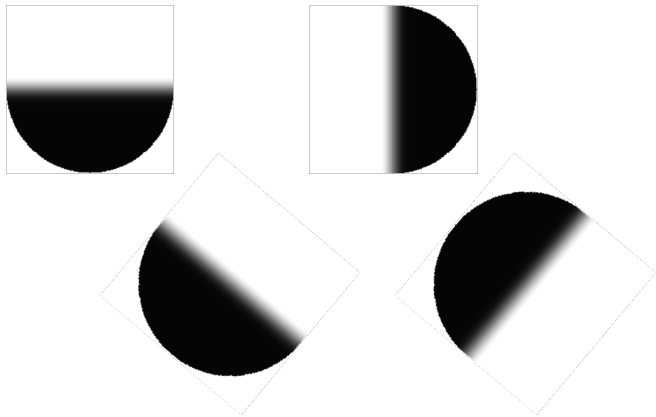
- gradient of image intensity
 $\nabla I = (\partial I / \partial x, \partial I / \partial y)$
- time derivative
 $\partial I / \partial t$
- velocity along gradient:
 v^\perp

- movement in direction of gradient:
 $v^\perp > 0$
- movement opposite direction of gradient:
 $v^\perp < 0$

$$v^\perp = - \frac{\partial I / \partial t}{|\nabla I|} = - \frac{\partial I / \partial t}{[(\partial I / \partial x)^2 + (\partial I / \partial y)^2]^{1/2}}$$

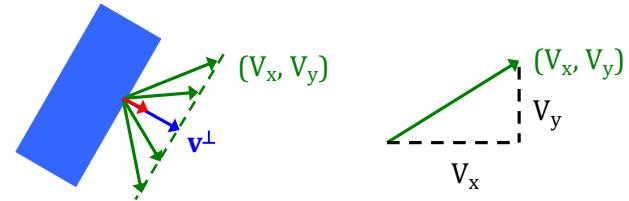
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Direction of the gradient $\nabla I = (\partial I / \partial x, \partial I / \partial y)$



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2-D velocities (V_x, V_y) consistent with v^\perp



All (V_x, V_y) such that the component of (V_x, V_y) in the direction of the gradient is v^\perp

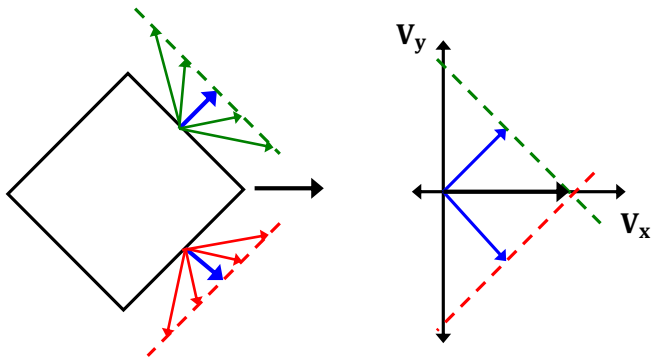
(u_x, u_y) : unit vector in direction of gradient

Use the dot product: $(V_x, V_y) \cdot (u_x, u_y) = v^\perp$

$$V_x u_x + V_y u_y = v^\perp$$

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Time-out exercise



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Details...

$$\begin{aligned} \partial I / \partial x &= 10 \\ \partial I / \partial y &= -10 \\ \partial I / \partial t &= -30 \end{aligned}$$

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$(V_x, V_y) ??$



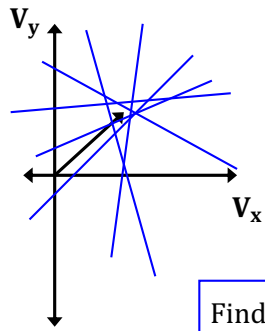
solve for V_x, V_y

For each component:

- (1) u_x
- (2) u_y
- (3) v^\perp
- (4) $V_x u_x + V_y u_y = v^\perp$

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In practice...



"least squares fit"

Previously...

$$V_x u_x + V_y u_y = v^\perp$$

New strategy:

Find (V_x, V_y) that **best fits** all motion components together

$$\text{Find } (V_x, V_y) \text{ that minimizes: } \sum (V_x u_x + V_y u_y - v^\perp)^2$$

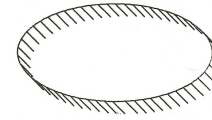
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Option 2: Smoothness assumption:

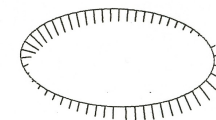
Compute a velocity field that:

- (1) is consistent with local measurements of image motion (perpendicular components)
- (2) has the *least amount of variation* possible

Pure Translation:



true & smoothest velocity field

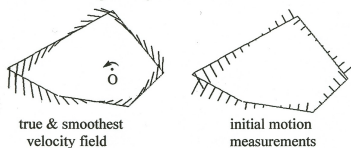


initial motion measurements

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When is the *smoothest* velocity field correct?

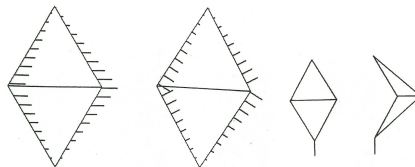
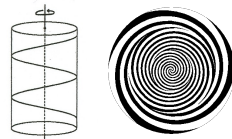
Rotation of rigid objects in 2D and 3D:



true & smoothest velocity field

initial motion measurements

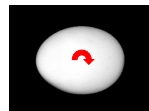
When is it wrong?



true & smoothest velocity field

initial motion measurements

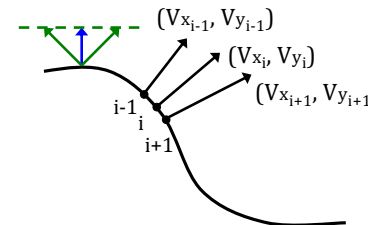
kinetic depth effect Wallach & O'Connell



motion *illusions*

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Computing the smoothest velocity field



motion components:

$$V_{x_i} u_{x_i} + V_{y_i} u_{y_i} = v^\perp_i$$

change in velocity:

$$(V_{x_{i+1}} - V_{x_i}, V_{y_{i+1}} - V_{y_i})$$

Find (V_{x_i}, V_{y_i}) that minimize:

$$\sum (V_{x_i} u_{x_i} + V_{y_i} u_{y_i} - v^\perp_i)^2 + \lambda [(V_{x_{i+1}} - V_{x_i})^2 + (V_{y_{i+1}} - V_{y_i})^2]$$

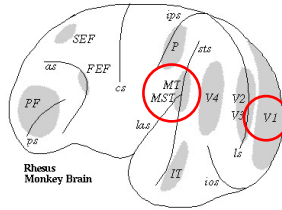
deviation from image motion measurements + variation in velocity field

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Two-stage motion measurement

motion components → 2D image motion

Movshon, Adelson, Gizzi & Newsome



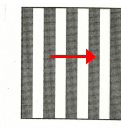
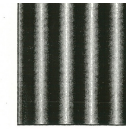
V1: high % of cells selective for direction of motion (especially in layer that projects to MT)

MT: high % of cells selective for direction and speed of motion

lesions in MT → behavioral deficits in motion tasks

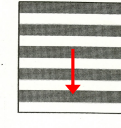
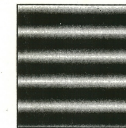
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Testing with sine-wave “plaids”

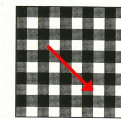


Moving plaid demo:

<http://www.georgemather.com/MotionDemos/PlaidMP4.html>

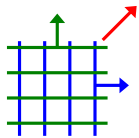


Movshon et al. recorded responses of neurons in area MT to moving plaids with different component gratings

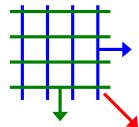


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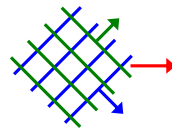
Logic behind the experiments



(1)



(2)



(3)

Component cells measure perpendicular components of motion

e.g. selective for vertical features moving right

predicted responses: (1) yes (2) yes (3) no

Pattern cells integrate motion components

e.g. selective for rightward motion of pattern

predicted responses: (1) no (2) no (3) yes

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Movshon et al. observations

- Cortical area V1:
 - all neurons behaved like component cells
- Cortical area MT:
 - layers 4 & 6: component cells
 - layers 2, 3, 5: pattern cells
- Perceptually, two components are not integrated if:
 - large difference in spatial frequency
 - large difference in speed
 - components have different stereo disparity

Evidence for two-stage motion measurement!

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