**CS230 Data Structures** **Handout # 29**
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# CS230 JEOPARDY: THE HOME VERSION

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## Asymptotics

**[1]** Consider the function $f(n) = 3n + 7$. List all of the following sets that do *not* contain $f$.

$$\begin{array}{ccc} O(1) & \Theta(1) & \Omega(1) \\ O(n) & \Theta(n) & \Omega(n) \\ O(n^2) & \Theta(n^2) & \Omega(n^2) \end{array}$$

**[2]** List the following eight asymptotic complexity classes in order from smallest to largest:

$$\begin{array}{cc} \Theta(n) & \Theta(1) \\ \Theta(3^n) & \Theta(n * \log(n)) \\ \Theta(\log(n)) & \Theta(n^2) \\ \Theta(n^3) & \Theta(2^n) \end{array}$$

**[3]** Give the asymptotic complexity class (i.e., $\Theta$ notation) for the best algorithm we have studied for each of the following problems:

**a.** Inserting $n$ elements, one at a time, into a sorted array.

**b.** Popping the top element off of a stack of $n$ elements.

**c.** Searching for an element in a balanced binary tree.

**c.** Sorting a list of $n$ elements.

**e.** Deleting an element from a balanced binary search tree.

**[4]** Match up each of the following algorithms with the recurrence equation that best characterizes it:

| | |
|---|---|
| binary search | $T_1(n) = 1 + T_1(n-1)$ |
| linear search | $T_2(n) = 1 + 2 * T_2(n-1)$ |
| merge sort | $T_3(n) = n + T_3(n-1)$ |
| selection sort | $T_4(n) = 1 + T_4(n/2)$ |
| towers of Hanoi | $T_5(n) = n + 2 * T_5(n/2)$ |

**[5]**
List *all* of the following recurrence equations whose solution is $\Theta(n)$.

$T_1(n) = 1 + T_1(n-1)$

$T_2(n) = 1 + 2 * T_2(n-1)$

$T_3(n) = n + T_3(n/2)$

$T_4(n) = n + T_4(n-1)$

$T_5(n) = n + 2 * T_5(n/2)$

$T_6(n) = 1 + T_6(n/2)$

$T_7(n) = 1 + 2 * T_7(n/2)$

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**Running Times**

**[1]**

Which of the following sorting algorithms have $\Theta(n * \log(n))$ running times in the worst case?

**a.** selection sort

**b.** merge sort

**c.** quick sort

**d.** insertion sort

**e.** heap sort (using a complete heap)

**f.** tree sort (i.e., build a BST and list elements in in-order)

**[2]**

List *all* of the following that can be done in linear time in the worst case.

**a.** Determining if $x$ is in a length-$n$ list.

**b.** Sorting a list of $n$ elements.

**c.** Building a BST from a list of $n$ elements.

**d.** Listing in sorted order all elements of an $n$-element BST.

**e.** Building a complete heap from a vector of $n$ elements.

**f.** Listing in sorted order all elements of an $n$-element complete heap.

**[3]**

List *all* of the following that can be done in logarithmic time in the worst case.

**a.** Determining if $x$ is in an array.

**b.** Determining if $x$ is in a sorted vector.

**c.** Determining if $x$ is in a balanced binary tree.

**d.** Determining if $x$ is in a balanced binary search tree.

**e.** Inserting $x$ into a complete heap.

**f.** Deleting $x$ from a leftist heap.

**g.** Merging two complete heaps.

**h.** Merging two leftist heaps.

**[4]**

Consider the following method:

```
public static IntList inOrderList (IntTree t) {
  if (IT.isLeaf) {
    return IL.empty();
  } else {
    return IL.append(inOrderList(IT.left(t)),
                     IL.prepend(IT.value(t),
                                inOrderList(IT.right(t))));
}
```

For each of the following assumptions, write a recurrence equation describes the worst running time of the method and give the asymptotic solution of the equation: (1) `t` is a balanced tree (2) `t` is an arbitrary tree.

**[5]**

(1) Write a recurrence equation for the worst case running time of the following method; (2) give the asympotic solution of the equation; and (3) describe a simple modification to the method that dramatically improves its asymptotic running time.

```
public static int f (ObjectList L) {
  if (IL.isEmpty) {
    return 0;
  } else {
    return IL.length(L)*(IL.length(L) + f(IL.tail(L)));
}
```

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## Gotchas

**[1]** What is the bug in the following Java method?

```
public int sum (Vector v) {
  int s = 0;
  for (int i = 0; i <= v.size(); i++) {
    s = s + v.get(i);
  }
  return s;
}
```

**[2]** What is the bug in the following Java class?

```
public class Location {

  private Point where;
```

```
   public Location (Point w) {
     Point where = w;
   }

   public int x () {
     return where.x;
   }

 }
```

[**3**] What is the bug in the following Java method?

```
public static int toInt (Object x) {
  if (x instanceof Integer) {
    return x.intValue();
  } else {
    return 0;
  }
}
```

[**4**] Draw a binary tree that is *not* a binary search tree but for which the following buggy isBST method returns `true`.

```
// *INCORRECT* version of isBST
public static boolean isBST (IntTree t) {
    return
      IT.isLeaf(t)
      || ((IT.isLeaf(IT.left(t)))
          || (IT.value(t) >= IT.value(IT.left(t))))
         && isBST(IT.left(t))
         && ((IT.isLeaf(IT.right(t)))
             || (IT.value(t) <= IT.value(IT.right(t))))
         && isBST(IT.right(t));
}
```

[**5**] What output is displayed by the following Java method when invoked on the following vector of strings: `[a, b, c, d, e]`?

```
public static void removeAndDisplay (Vector v) {
  for (int i = 0; i < v.size(); i++) {
    System.out.println(v.remove(i));
  }
}
```

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**Lists**

[**1**] Several data structure implementations we have studied have instance variables that are mutable lists with dummy header nodes. What is the purpose of the dummy header node?

[**2**] What is wrong with the following method for testing if an integer list is sorted?

```
public static boolean isSorted (IntList L) {
  return (IL.isEmpty(L))
         || ((IL.head(L) <= IL.head(IL.tail(L)))
             && (isSorted(IL.tail(L))));
  }
}
```

**[3]** Describe (1) one advantage of a mutable list over an immutable list *and* (2) one advantage of an immutable list over a mutable list.

**[3]** Describe (1) one advantage of a mutable list over an immutable list *and* (2) one advantage of an immutable list over a mutable list.

**[4]** Flesh out the body of the following method for turning a *non-empty*, *non-cyclic*, *mutable* list into a cyclic list (i.e., a list whose last node's tail points to its first node).

```
public static void cylcify (ObjectMList L) {
  // flesh this out;
}
```

**[5]** Answer both of the following: (1) what is the running time of the following method on a list of length $n$? (2) rewrite the body of the method so that it has the same behavior but has an asymptotically better running time.

```
public static IntList revDouble (IntList L) {
  if (IL.isEmpty(L)) {
    return L;
  } else {
    IL.postpend(revDouble(IL.tail(L)),
                2*IL.head(L));
  }
}
```

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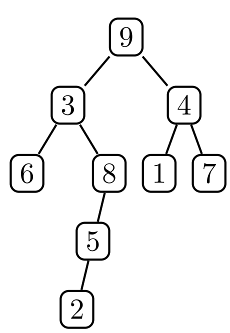
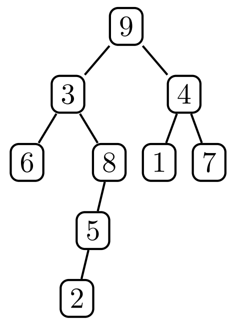
**Trees**

**[1]** List the values of all nodes in the following tree at which the heap condition is *not* satisfied.



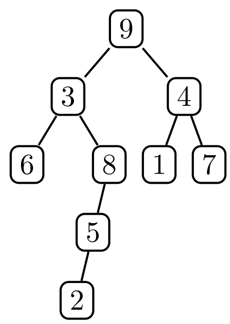
**[2]**

List the values of all nodes in the following tree at which the binary search tree condition is *not* satisfied.



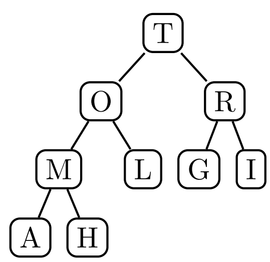

**[3]**

List the values of all nodes in the following tree at which the leftist condition is *not* satisfied.




**[4]**

Draw the tree that results from dequeuing an element from the following complete heap:




**[5]** For every $n$, is it possible that all the distinct integers in $[1..n]$ can be arranged into a single binary tree that is both a BST and a leftist heap? Assume that larger numbers have higher priority.

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**Potpourri**

**[1]** List all of the following collections for which a balanced binary search tree is an efficient representation:

- bag

- priority queue
- queue
- set
- stack
- table

**[2]**

For each of the following real-life collections, give the data structure that would most appropriately model the situation:

| | |
|---|---|
| cities visited during a trip | bag |
| dictionary entries | priority queue |
| emergency room waiting area | queue |
| pile of papers | set |
| shopping cart of grocery items | stack |
| supermarket checkout line | table |

**[3]**

Match up each of the following collections with the data structure that is most appropriate for representing it.

| | |
|---|---|
| priority queue | 2-3 tree |
| queue | cyclic linked list |
| sequence | leftist heap |
| set | non-cyclic linked list |
| stack | vector |

**[4]** Answer *both* of the following:

1. Is every complete heap a leftist heap?
2. Is every leftist heap a complete heap?

**[5]** What is the largest $n$ such that all the distinct integers in $[1..n]$ can be arranged into a single binary tree that is both a BST and max complete heap?