CS231 Algorithms
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Handout # 28
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Matrix-Chain Multiplication

Reading: *CLRS* Ch. 15.2, *CLR* Ch. 16.1

Cost of Dynamic Programming

In general the cost of dynamic programming is:

$$(\text{number of table slots}) \times (\text{cost to fill in each table slot})$$

- In the examples studied thus far (exponentiation, Fibonacci numbers, Pascal's triangle, longest common subsequence), the cost to fill a table slot has been constant, so the cost of the algorithm is proportional to the number of slots.
- In general, the cost of filling slots may be non-trivial. Here we consider one such problem: **matrix-chain multiplication**.

Matrix-Chain Multiplication

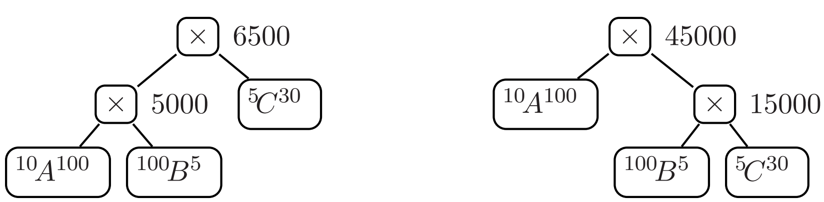
Let the notation ${}^{r}M^{c}$ denote a two dimensional matrix of numbers with r rows and c columns. Recall the following facts about matrix multiplication:

1. The the multiplication ${}^{a}M_1^{b} \times {}^{c}M_2^{d}$ is defined only if $b = c$, in which case the result is a matrix ${}^{a}M^{d}$.
2. The cost of the multiplication ${}^{a}M_1^{b} \times {}^{b}M_2^{c}$ is abc.
3. Matrix multiplication is associative. I.e.,

$${}^{a}M_1^{b} \times ({}^{b}M_2^{c} \times {}^{c}M_3^{d}) = ({}^{a}M_1^{b} \times {}^{b}M_2^{c}) \times {}^{c}M_3^{d}$$

We say that a sequence ${}^{r_1}M_1^{c_1} \times \ldots \times {}^{r_i}M_i^{c_i} \times \ldots {}^{r_n}M_n^{c_n}$ is **compatible** if $c_i = r_{i+1}$ for all i in $1 \leq i < n$.

The cost of a compatible sequence of matrix multiplications depends on its **schedule**, the particular multiplication tree used. For example:



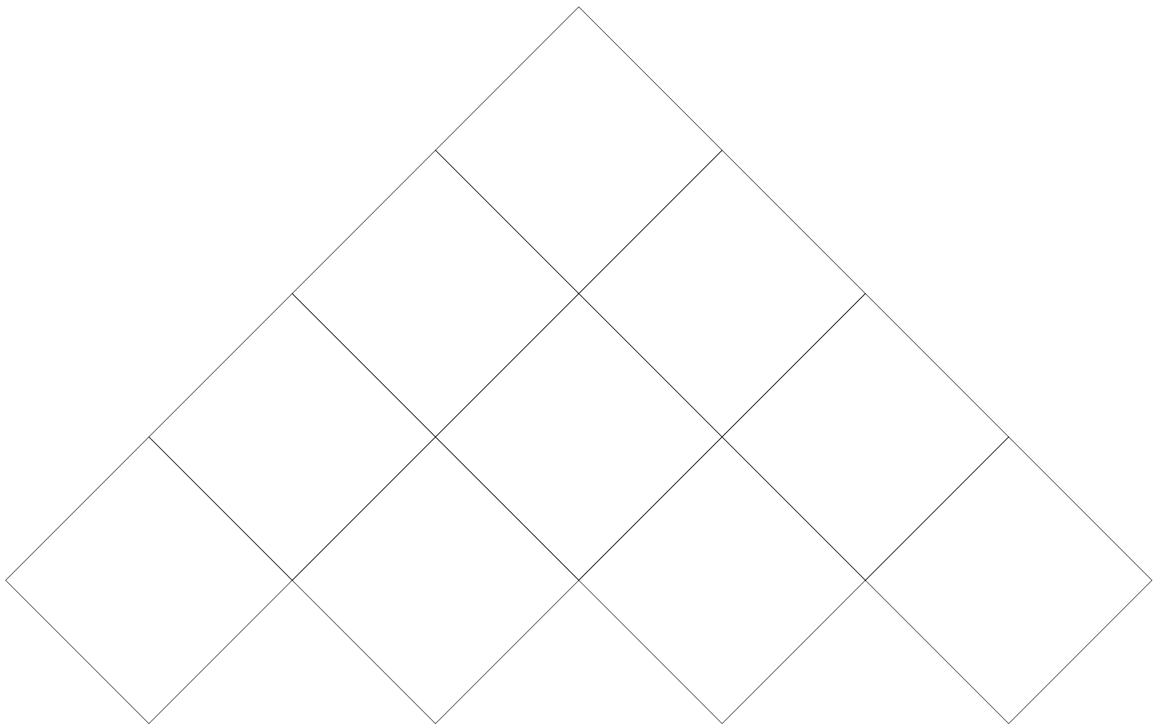
The **matrix-chain multiplication problem** is:

Given a length-n compatible sequence $\prod_{i=1}^{n} {}^{r_i}M_i^{c_i}$, find the schedule with least cost.

Dynamic Programming Solution

Idea: For each contiguous subsequence of matrices, determine the optimal schedule and its cost. This can be done by considering all possible splits of the subsequence into two smaller subsequences.

Example: Determine the optimal schedule for ${}^{10}A^{100} \times {}^{100}B^{5} \times {}^{5}C^{30} \times {}^{30}D^{20}$



What is the cost of the dynamic programming solution for a sequence of n matrices?