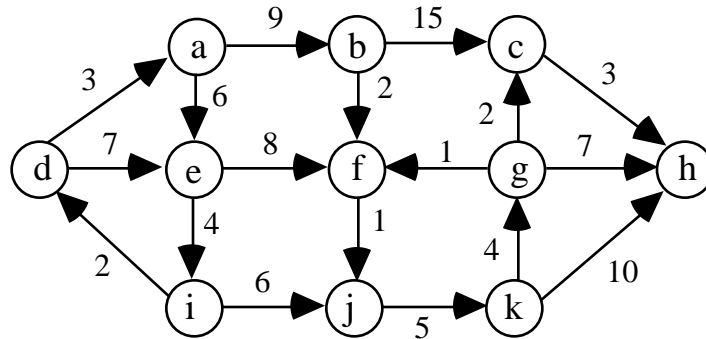


PROBLEM SET 7**Due: Monday, November 25**

Reading: Handouts 30 & 31; CLR Section 5.4 (Graphs); Chapter 23 (Elementary Graph Algorithms); Chapter 24 (Minimum Spanning Trees); Sections 25.1 -- 25.2 (Relaxation + Dijkstra's algorithm). I suggest you skim all the sections and look at the figures but avoid reading the proofs. Note that some of the graph algorithms I presented in class are somewhat different than the ones presented in the book.

Suggested Problems: 23.1-1; 23.2-6; 23.3-9; 23.4-2, 23.4-5; 23.5-1, 23.5-3, 23.5-4, 23.5-4; 23-1, 23-2; 24.1-7, 24.2-4, 24.2-5, 25.2-1, 25.2-3, 25.2-5, 25.2-6

Problem 1 [50] Consider the following weighted, directed graph G :



In the following problems, you should assume that G is represented as a collection of adjacency lists, and that vertices are ordered alphabetically within each adjacency list.

a [5] Draw the tree that is induced by performing breadth-first search starting at node d .

b [15] Use Dijkstra's algorithm to determine the shortest paths from d to each of the other vertices. Draw the shortest path tree that is induced by running Dijkstra's algorithm, and annotate each vertex by its shortest path distance from d .

c [10] Consider the undirected graph G' that is the result of erasing the directions on each of the edges in G . Draw a minimum spanning tree of G' .

d [10] Draw the tree that is induced by performing a depth-first-search starting at node d . Label each vertex by its discovery and finishing times (as on CLR p. 479).

e [5] Give a topological sort of the vertices in G .

f [5] Draw a copy of G and circle the strongly-connected components of G .

Problem 2 [10] (A modified version of CLR 25.2-2, p. 531)

Dijkstra's algorithm assumes that the edge weights of the given graph are non-negative. Here we consider some of the consequences of allowing non-negative edge weights.

a [5] A **negative-weight cycle** of a weighted graph is a cyclic path whose path weight is negative. Suppose that G is a directed graph with a negative-weight cycle. Explain why the single-source shortest path problem may not be well-defined on such a graph.

b [5] If a graph G has some edges with negative weights but no negative-weight cycles, then the single-source shortest path problem is still well-defined. However, Dijkstra's algorithm is not guaranteed to correctly solve the problem in the presence of negative edge weights. Construct a connected, directed, weighted graph with four vertices such that Dijkstra's algorithm gives an incorrect solution to the single-source shortest path problem.

Problem 3 [15]

a [10] CLR 23.4-3 (p. 488).

b [5] Does your algorithm from part a work for directed graphs? Explain.

Problem 4 [15] CLR 23.4-5 (p. 488)

Extra Credit Problem [20] CLR 24.2-8 (p. 510)

Problem Set Header Page
Please make this the first page of your hardcopy submission.

CS231 Problem Set 7
Due Monday, November 25, 1996

Name:

Date & Time Submitted (*only if late*):

Collaborators (*anyone you collaborated with in the process of doing the problem set*):

*In the **Time** column, please estimate the time you spent on the parts of this problem set. Please try to be as accurate as possible; this information will help me to design future problem sets. I will fill out the **Score** column when grading your problem set.*

Part	Time	Score
General Reading		
Problem 1 [50]		
Problem 2 [10]		
Problem 3 [15]		
Problem 4 [15]		
Extra Credit [20]		
Total		