Announcements

● Change in deadlines
  ○ Assignment 4 is due on March 11th
  ○ Assignment 5 is due on March 18th
  ○ Schedule gets back to normal after Spring Break

● Discussion
  ○ Wednesday, 6-7pm (Sappha)

● Drop-in hours
  ○ Sunday, 6-8pm (Hunter)
  ○ Monday, 6-8pm (Alison)
  ○ Wednesday, 7-9pm (Denise)

● Office hours
  ○ Monday/Friday
Interval Scheduling (cont.)

Reading: Section 4.1 and 4.2
Problem 1

- Set of $n$ jobs to be executed
- Each job has a start time and a finish time: $s_1, \ldots, s_n, f_1, \ldots, f_n$
- Jobs cannot run in parallel
  - Jobs are compatible if their time does not overlap

- **Goal:** Find maximum set of compatible jobs
Algorithm 1

Initially let $R$ be the set of all requests, and let $A$ be empty

While $R$ is not yet empty

Choose a request $i \in R$ that has the smallest finishing time

Add request $i$ to $A$

Delete all requests from $R$ that are not compatible with request $i$

EndWhile

Return the set $A$ as the set of accepted requests
Correctness

- Stay-ahead argument: after each step of the greedy algorithm, its solution is at least as good as any optimal one.

- Optimality of algorithm 1
  - Let $A$ be the set returned by the algorithm and consider an optimal solution $B$.
  - Show that $A$ is a set of compatible jobs.
  - Show that $|A| = |B|$.
Show that $|A| = |B|$

Let $A = \{i_1, i_2, \ldots, i_k\}$ and $B = \{j_1, j_2, \ldots, j_m\}$.

**Lemma:** For all indices $r \leq k$, $f(i_r) \leq f(j_r)$. (Stay-ahead)

Proof by induction.

**Lemma:** Greedy solution is optimal.

By contradiction, using (1), show $|A| = |B|$. 

Problem 2

- Set of $n$ jobs to be executed
- Each job has a deadline and an execution time: $d_1, \ldots, d_n, t_1, \ldots, t_n$
- Jobs cannot run in parallel
- All jobs must be executed

**Goal:** Find scheduling that minimizes maximum lateness
Algorithm 2

Order the jobs in order of their deadlines
Assume for simplicity of notation that $d_1 \leq \ldots \leq d_n$
Initially, $f = s$
Consider the jobs $i = 1, \ldots, n$ in this order
   Assign job $i$ to the time interval from $s(i) = f$ to $f(i) = f + t_i$
   Let $f = f + t_i$
End
Return the set of scheduled intervals $[s(i), f(i)]$ for $i = 1, \ldots, n$
Correctness

- Exchange approach: Transform any solution into a greedy solution without making it worse.

- Optimality of algorithm 2
  - There is an optimal solution with no idle time.
  - All schedules with no inversions and no idle time have the same maximum lateness.
  - There is an optimal schedule that has no inversions and no idle time.
  - Greedy solution has no inversions.
Idle Time

There is an optimal solution with no idle time.
Inversions

Def.: Consider a schedule of the jobs. This schedule has an inversion if for some jobs, we have $i < j$ and $d_j < d_i$.

Greedy solution has no inversions!

Lemma: All schedules with no inversions and no idle time have the same maximum lateness.
Optimal Solution without Inversion

Lemma: There is an optimal schedule that has no inversions and no idle time.

Proof: Let B be an optimal solution. Suppose B has some inversion $i, j$.

- Swapping jobs $i$ and $j$ creates a schedule with one less inversion
- The maximum lateness does not get worse.