Lecture 11 – Interval Scheduling
Reading: KT Section 4.2

Partial content of these slides have been obtained from the official lecture slides that accompany the textbook. A complete set of slides can be found at: http://www.cs.princeton.edu/~wayne/kleinberg-tardos/

Scheduling to minimize lateness

The exchange argument approach
Scheduling to minimize lateness

• Setting:
  • A single resource
  • Each job j requires \( t(j) \) time units, and a deadline of \( d(j) \)
  • \( j \) starts at \( s(j) \), and is done at \( f(j) = s(j) + t(j) \)
  • The lateness of a job \( j \) is \( l(j) = f(j) - d(j) \) if it passes the deadline
    • \( \text{And} \ 0 \ \text{otherwise} \)

• Goal:
  • Decide on the start time of each job to minimize the maximum lateness \( (L) \)
  • \( L = \max_j l(j) \)

Greedy algorithms to minimize lateness

• How would you order the jobs in the schedule?
Earliest deadline first

**Earliest-Deadline-First** \((n, t_1, t_2, \ldots, t_n, d_1, d_2, \ldots, d_n)\)

- **Sort** \(n\) jobs so that \(d_1 \leq d_2 \leq \ldots \leq d_n\).
- \(t \leftarrow 0\)
- **For** \(j = 1\) to \(n\)
  - Assign job \(j\) to interval \([t, t + t_j]\).
  - \(s_j \leftarrow t\)
  - \(f_j \leftarrow t + t_j\)
  - \(t \leftarrow t + t_j\)
- **Return** intervals \([s_1, f_1]\), \([s_2, f_2]\), \ldots, \([s_n, f_n]\).

There is an optimal schedule that has no inversions and no idle time.

**Correctness – Exchange approach**

**What is it?**
- We start with an optimal solution \(O\), and turn it into the greedy solution \(A\).

**For the earliest deadline first algorithm...**
- There exists an optimal schedule with no idle time.
- The earliest-deadline-first schedule has no idle time.
- The earliest-deadline-first schedule has no inversions.
- All schedules with no inversions and no idle time have the same maximum lateness.
- If a schedule (with no idle time) has an inversion, it has one with a pair of inverted jobs scheduled consecutively.
- Swapping two adjacent, inverted jobs reduces the number of inversions by one and does not increase the max lateness.

**There is an optimal schedule that has no inversions and no idle time.**

**The schedule A produced by the greedy algorithm has optimal maximum lateness** \(L\).