Lecture 7 – Graphs
Reading: KT Sections 3.1 and 3.2

Let’s start with some basics

Graph representation
Undirected Graphs

- Undirected graph. $G = (V, E)$
- $V =$ nodes.
- $E =$ edges between pairs of nodes.
- Captures pairwise relationship between objects.
- Graph size parameters: $n = |V|, m = |E|.$

$V = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$E = \{1-2, 1-3, 2-3, 2-4, 2-5, 3-7, 3-8, 4-5, 5-6\}$
$n = 8$
$m = 11$

Some Graph Applications

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World Wide Web

- Web graph.
  - Node: web page.
  - Edge: hyperlink from one page to another.

9-11 Terrorist Network

- Social network graph.
  - Node: people.
  - Edge: relationship between two people.

Ecological Food Web

- Food web graph.
  - Node = species.
  - Edge = from prey to predator.


Paths and Connectivity

- Def. A path in an undirected graph $G = (V, E)$ is a sequence $P$ of nodes $v_1, v_2, ..., v_k$ with the property that each consecutive pair $v_i, v_{i+1}$ is joined by an edge in $E$.

- Def. A path is simple if all nodes are distinct.

- Def. An undirected graph is connected if for every pair of nodes $u$ and $v$, there is a path between $u$ and $v$. 
Cycles

- **Def.** A **cycle** is a path $v_1, v_2, ..., v_{k-1}, v_k$ in which $v_1 = v_k$, $k > 2$, and the first $k-1$ nodes are all distinct.

![](cycle.png)

_cycle $C = 1-2-4-5-3-1_

Trees

- **Def.** An undirected graph is a **tree** if it is connected and does not contain a cycle.

- **Theorem.** Let $G$ be an undirected graph on $n$ nodes. Any two of the following statements imply the third.
  - $G$ is connected.
  - $G$ does not contain a cycle.
  - $G$ has $n-1$ edges.
Rooted Trees

• Rooted tree. Given a tree T, choose a root node r and orient each edge away from r.

• Importance. Models hierarchical structure.

Exercise time!

Graph representation
Graph Representation: Adjacency Matrix

- Adjacency matrix. n-by-n matrix with $A_{uv} = 1$ if $(u, v)$ is an edge.
  - Two representations of each edge.
  - Space proportional to __________.
  - Checking if $(u, v)$ is an edge takes $\Theta(?)$ time.
  - Identifying all edges takes $\Theta(?)$ time.

```
1 2 3 4 5 6 7 8
1 0 1 1 0 0 0 0 0
2 1 0 1 1 0 0 0 0
3 1 1 0 0 1 0 1 1
4 0 1 0 1 0 0 0 0
5 0 1 1 0 1 0 0 0
6 0 0 0 0 1 0 0 0
7 0 0 1 0 0 0 0 1
8 0 0 1 0 0 0 1 0
```

Graph Representation: Adjacency List

- Adjacency list. Node indexed array of lists.
  - Two representations of each edge.
  - Space proportional to __________.
  - Checking if $(u, v)$ is an edge takes $O(?)$ time.
  - Identifying all edges takes $\Theta(?)$ time.

```
1 2 3 4 5 6 7 8
1 1 2 5 4
2 1 2 3
3 2 3 5 8
4 2 4
5 5 4
6 6
7 7
8 8
```