Exercise 3-4. Use the iteration method to solve the following recurrences. Assume $T(n) = 1$ for $n < 2$. Express your solutions in $\Theta$ notation. Show your work. (Note that parts b through e generalize examples discussed in class.)

a. $T(n) = 4T(n/2) + n$

b. $T(n) = T(n-k) + 1$, $k > 0$

c. $T(n) = T(n-k) + n$, $k > 0$

d. $T(n) = T(an) + 1$, $0 < a < 1$

e. $T(n) = T(an) + n$, $0 < a < 1$

f. $T(n) = T(an) + T(bn) + n$, $0 < (a+b) < 1$

CLRS Exercise 6.1-1. What are the minimum and maximum numbers of elements in a heap of height $h$? (Hint: See CLRS Appendix B.)

CLRS Exercise 6.1-2. Show that an $n$-element heap has height $\lceil \lg n \rceil$. (Hint: Use Exercise 6.1-1.)

CLRS Exercise 6.1-7. Show that, with the array representation for storing an $n$-element heap, the leaves are the nodes indexed by $\lceil n/2 \rceil+1$, $\lceil n/2 \rceil+2$, $\ldots$, $n$.

CLRS Problem 6.1. The procedure BUILD-MAX-HEAP in Section 6.3 can be implemented by repeatedly using Max-Heap-Insert to insert the elements into the heap. Consider the following implementation:

BUILD-MAX-HEAP'(A)
1 $A.heap-size = 1$
2 for $i = 2$ to $A.length$
3 MAX-HEAP-INSERT($A, A[i]$)

a. Do the procedures BUILD-MAX-HEAP and BUILD-MAX-HEAP' always create the same heap when run on the same input array? Prove that they do, or provide a counterexample.
b. Show that in the worst case, BUILD-MAX-HEAP' requires $\Theta (n \lg n)$ time to build an $n$-element heap.