Fundamental Algorithms

Asymptotic Notation

CLRS Reading: Chapter 3, pages 41--61, Appendix A, pages 1058--1069
Problem Set: Assignment #1 due Friday, February 8

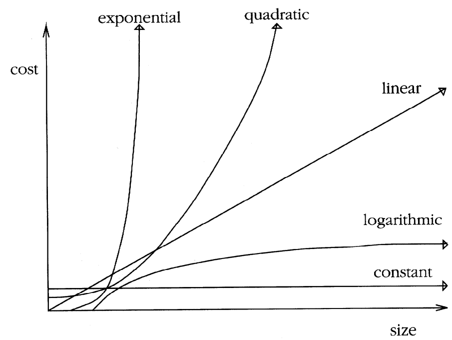
Fundamental Algorithms

What a Mess!

InsertionSort(*A*)	***cost***	***times***
for *j* ← 2 to *length*[*A*]	1	*n*
do *key* ← *A*[*j*]	1	*n*-1
» Insert *A*[*j*] into the sorted sequence *A*[1..*j*-1]	0	*n*-1
i ← *j*-1	1	*n*-1
while *i* > 0 and *A*[*i*] > *key*	1	*n*(*n*+1)/2-1
do *A*[*i*+1] ← *A*[*i*]	1	*n*(*n*-1)/2
i ← *i*-1	1	*n*(*n*-1)/2
A[*i*+1] ← *key*	1	*n*-1

$T(n) = (1/2 + 1/2 + 1/2)n^2 + (1 + 1 + 1 + 1/2 - 1/2 - 1/2 + 1)n + (-1 - 1 - 1 - 1)$

The Way Things Grow



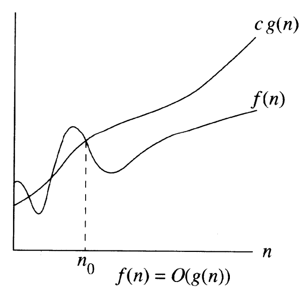
C - 3

No Faster Than...

Definition

$f = O(g)$ if $\exists$ positive constants c, n_0

such that $\forall\ n \geq n_0\ 0 \leq f(n) \leq c\,g(n)$



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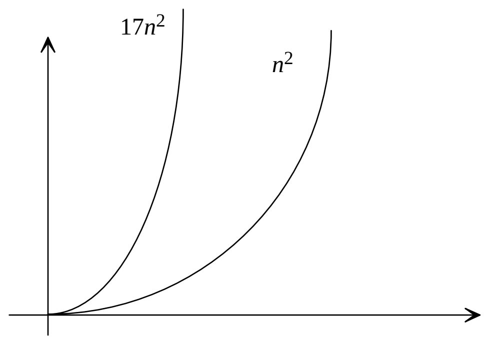
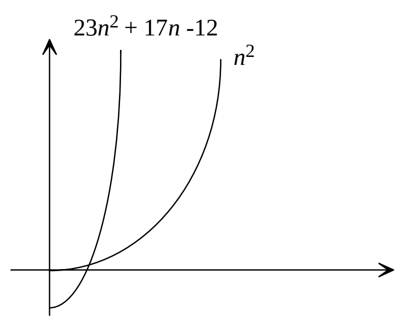
$17n^2$ grows no faster than n^2



We say that $f = O(g)$ if $\exists$ positive constants c, n_0 such that $\forall\ n \geq n_0$ $0 \leq f(n) \leq c\,g(n)$.
What are n_0 and c in this case?

C - 5

$23n^2 + 17n - 12 = O(n^2)$



Recall $f = O(g)$ if $\exists$ positive constants c, n_0 such that $\forall\ n \geq n_0$ $0 \leq f(n) \leq c\,g(n)$.
There are many pairs (c, n_0) to choose from.

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Fundamental Algorithms

Is $n^3 = O(n^2)$?

If it were, then there would exist constants c and n_0 so that for all $n \geq n_0$, $n^3 \leq c\,n^2$. *

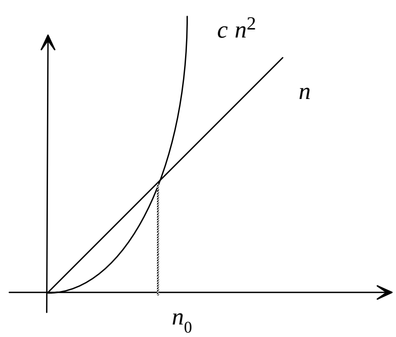
Hmmm...

*This style of argument is known as a proof by contradiction.

Fundamental Algorithms

Strictly Slower Than...

$f = o(g(n))$ if $\forall$ positive constants c, $\exists\ n_0 > 0$ such that $\forall\ n \geq n_0\ 0 \leq f(n) \leq c\,g(n)$



Compare: $f = O(g)$ if $\exists$ positive constants c, n_0 such that $\forall\ n \geq n_0\ 0 \leq f(n) \leq c\,g(n)$.

Is *ln n* = *o*(√*n*)?

Remark

$f = o(g)$ if and only if $\lim_{n\to\infty} f(n)/g(n) = 0$

Thus,

we need only find $\lim_{n\to\infty} \ln n/\sqrt{n}$

Recall $f = o(g)$ if $\forall$ positive constants c, $\exists\ n_0 \geq 0$ such that $\forall\ n \geq n_0\ 0 \leq f(n) \leq c\, g(n)$.
Is $lg\ n = o(\sqrt{n})$?

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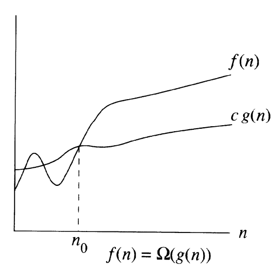
No Slower Than...

Definition

$f = \Omega(g)$ if $\exists$ positive constants c, n_0

such that $\forall\ n \geq n_0\ 0 \leq c\, g(n) \leq f(n)$



$f(n) = \Omega(g(n))$

*Yes, there is a strictly faster than. It is denoted $f = \omega(g)$ and you can read about it in the text.

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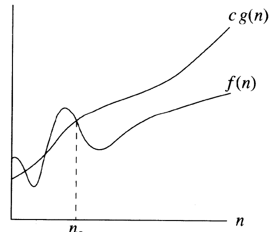
A Rose By Any Other Name

Compare

$f = O(g)$ if $\exists$ positive constants c, n_0 such that $\forall\ n \geq n_0$ $0 \leq f(n) \leq c\,g(n)$



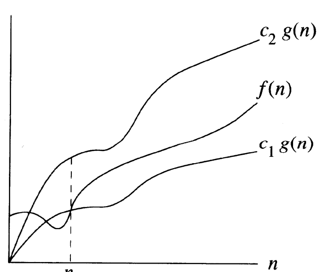
$f(n) = O(g(n))$

$g = \Omega(f)$ if $\exists$ positive constants c, n_0 such that $\forall\ n \geq n_0$ $0 \leq c\,f(n) \leq g(n)$

Just Right*

Definition

$f = \Theta(g)$ if $\exists$ positive constants c_1, c_2, n_0 such that $\forall\ n \geq n_0$ $0 \leq c_1\,g(n) \leq f(n) \leq c_2\,g(n)$



$f(n) = \Theta(g(n))$

*Thus, $f = \Theta(g)$ if and only if $f = O(g)$ and $f = \Omega(g)$.

Is $23n^2 + 17n - 12 = \Theta(n^2)$?

Remark

If $\lim_{n\to\infty} f(n)/g(n) = k > 0$ then $f = \Theta(g)$*.

Thus,

if $\lim_{n\to\infty} (23n^2 + 17n - 12)/n^2 = k > 0$, …

*The converse fails. For example, $f(n) = 2 + sin(n)$ and $g(n) = 2$.

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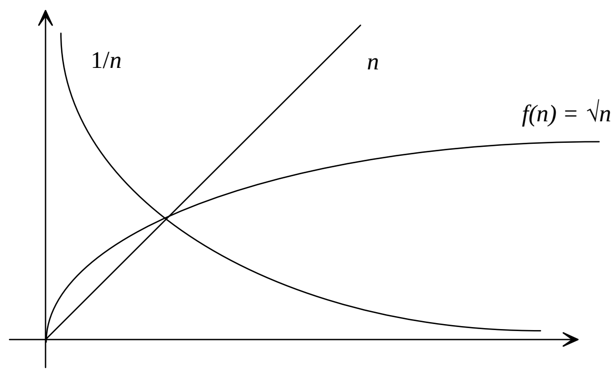
Summary

Say that f is	Mean that f is	Write	If
small oh g	slower than g	$f = o(g)$	$\lim_{n\to\infty} f(n)/g(n) = 0$
oh g	no faster than g	$f = O(g)$	$\exists\, c, n_0 > 0: \forall\, n > n_0, f(n) \le c\, g(n)$
theta g	about as fast as g	$f = \theta(g)$	$f = O(g)$ and $g = O(f)$
omega g	no slower than g	$f = \Omega(g)$	$g = O(f)$
small omega g	faster than g	$f = \omega(g)$	$g = o(f)$

C - 14

Where is $g(n) = n^{\sin(n)}$?



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