

Quicksort

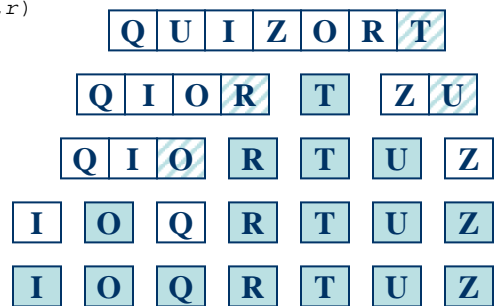
CLRS Reading: Sections 7.1, 7.2, pages 145 -- 153
 Problem Set: Assignment #3 due Friday, February 22

G-1

Quicksort

```

Quicksort( $A, p, r$ )
  if  $p < r$ 
    then  $q \leftarrow \text{Partition}(A, p, r)$ 
         Quicksort( $A, p, q-1$ )
         Quicksort( $A, q+1, r$ )
    
```

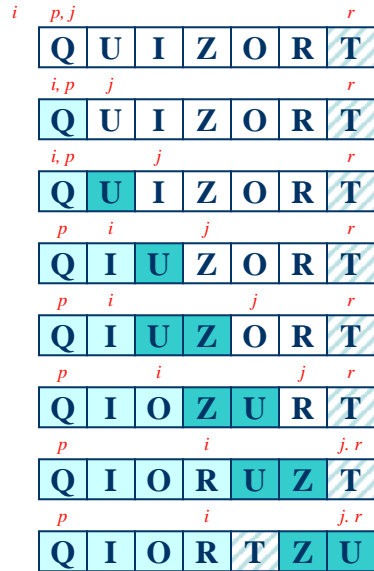


G-2

Partitioning

```

Partition(A, p, r)
  x ← A[r]    {serves as pivot}
  i ← p-1
  for j ← p to r-1
    do if A[j] ≤ x
      then i ← i+1
           exchange A[i] ↔ A[j]
  exchange A[i+1] ↔ A[r]
  return i+1
    
```



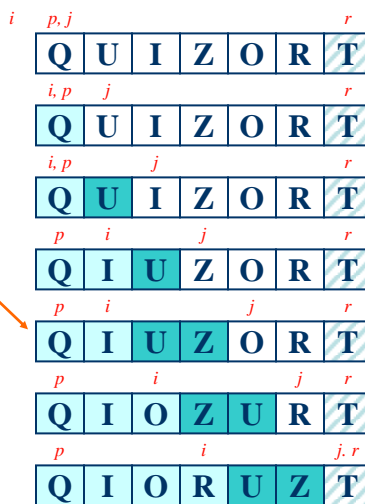
G-3

At the beginning of each iteration,

1. If $p \leq k \leq i$, then $A[k] \leq x$.

2. If $i+1 \leq k \leq j-1$, then $A[k] > x$.

3. If $k = r$, then $A[k] = x$.



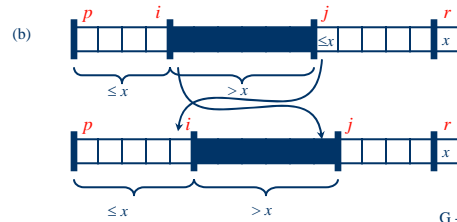
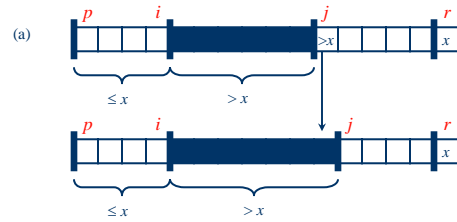
*Remember, we must show *initialization*; *maintenance*; and *termination*.

G-4

Maintaining Loop Invariant

```

Partition(A, p, r)
  x ← A[r]
  i ← p-1
  for j ← p to r-1
    do if A[j] ≤ x
      then i ← i+1
         exchange A[i] ↔ A[j]
  exchange A[i+1] ↔ A[r]
  return i+1
  
```



G-5

Running Time of Partition

```

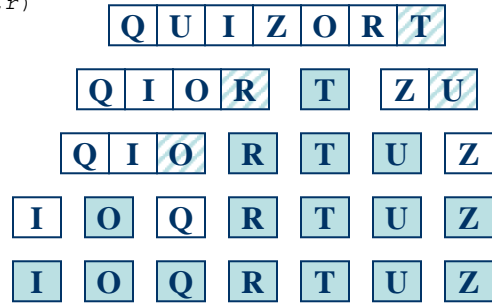
Partition(A, p, r)
1  x ← A[r]    {serves as pivot}
2  i ← p-1
3  for j ← p to r-1
4    do if A[j] ≤ x
5      then i ← i+1
6         exchange A[i] ↔ A[j]
7  exchange A[i+1] ↔ A[r]
8  return i+1
  
```

G-6

Correctness of Quicksort*

```

Quicksort(A, p, r)
  if p < r
    then q ← Partition(A, p, r)
         Quicksort(A, p, q-1)
         Quicksort(A, q+1, r)
  
```

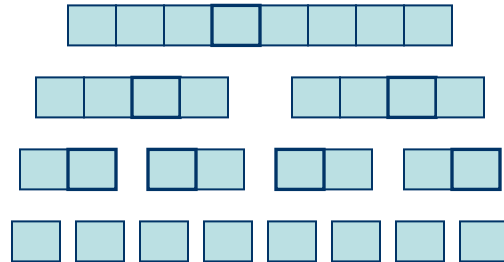


*By induction on the size of the subproblem, using correctness of Partition algorithm.

G - 7

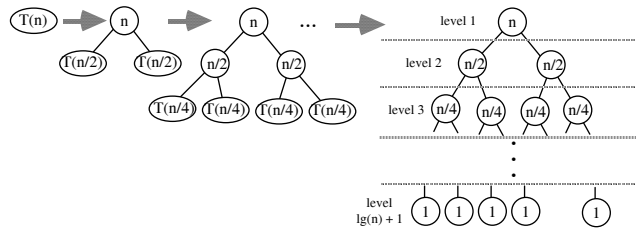
The Best

- Intuitively, the best we can hope for is a balanced partition.



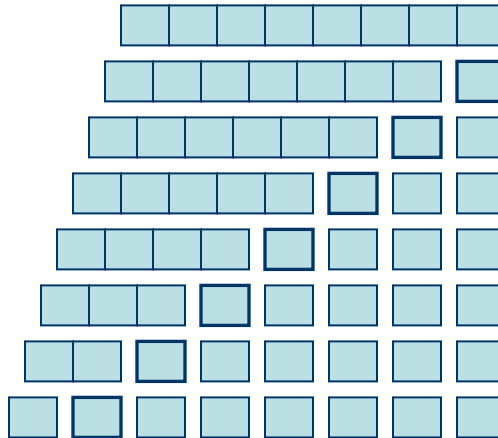
G - 8

$$T(n) = 2 * T(n/2) + n$$



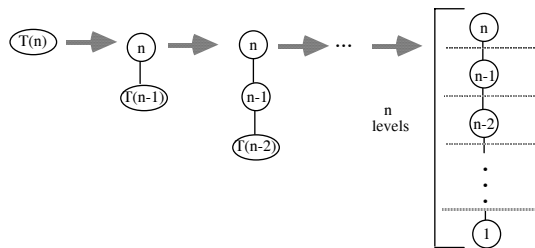
The Worst

- Intuitively, the worst we can imagine is a totally unbalanced partition:



*Could such a thing actually happen?

$$T(n) = T(n-1) + n$$



Making the Worst Case Less Likely

```

Randomized-Partition( $A, p, r$ )
   $i \leftarrow \text{Random}(p, r)$ 
  exchange  $A[r] \leftrightarrow A[i]$ 
  return Partition( $A, p, r$ )
  
```

```

Randomized-Quicksort( $A, p, r$ )
  if  $p < r$ 
    then  $q \leftarrow \text{Randomized-Partition}(A, p, r)$ 
         Randomized-Quicksort( $A, p, q-1$ )
         Randomized-Quicksort( $A, q+1, r$ )
  
```