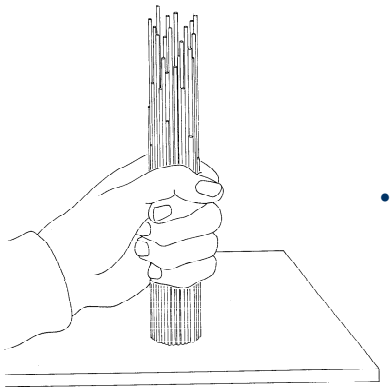
Limits to Comparison Sorting

CLRS Reading: Section 8.1, pages 165 -- 168
Problem Set: Assignment #4 due Friday, February 29

How Good a Sort Can One Expect?

Algorithm	Worst	Comments
Insertion Sort	n^2	Best for nearly sorted files
Quicksort	n^2	Very fast on average
Mergesort	$n \log n$	Best worst case

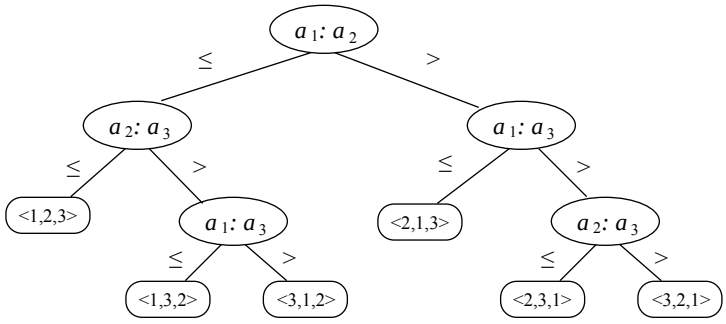
Models of Computation



- If we know something about the structure of the file being sorted, then special case sorts can perform miracles.
- What model of complexity are we using? *Spaghetti sort*, for example, has a fast worst-case run-time, but would be difficult to implement in Java.

Decision Tree Model

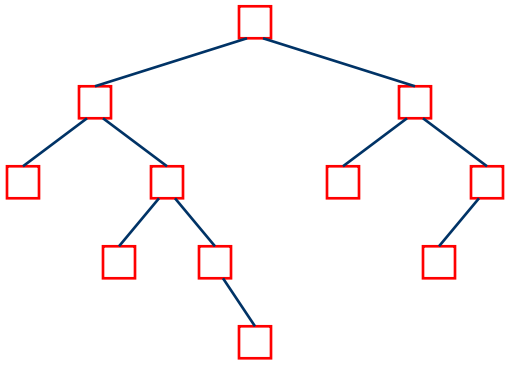
- We consider algorithms in which the only operation used to gain information about the sequence is comparison of two elements.



Leaves and Branches

Lemma

A binary tree of height h has at most 2^h leaves.



Corollary

A binary tree with n leaves must be at least lg n high.

Limits to Comparison Sorting

Theorem 9.1

Any decision tree that sorts n elements has height $\Omega(n \lg n)$.

Corollary 9.2

Heapsort and merge sort are asymptotically optimal comparison sorts.