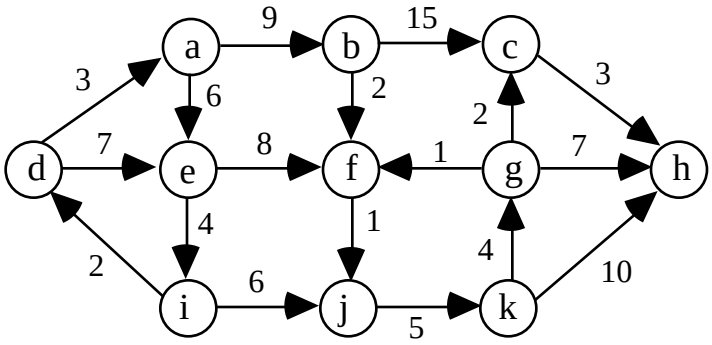
CS231 Algorithms **Handout #27**
Prof Lyn Turbak **April 19, 2001**
Wellesley College

PROBLEM SET 8
Due: Thursday, April 26

Reading: Handouts 25--26; CLR Section 5.4 (Graphs); Sections 23.1--23.2 (Bread-First Search); Chapter 24 (Minimum Spanning Trees); Sections 25.1 -- 25.2 (Relaxation + Dijkstra's algorithm). I suggest you skim all the sections and look at the figures but avoid reading the proofs. Note that some of the graph algorithms I presented in class are somewhat different than the ones presented in the book.

Suggested Problems: 23.1-1; 24.1-7, 24.2-4, 24.2-5, 25.2-1, 25.2-3, 25.2-5, 25.2-6

Problem 1 [50] Consider the following weighted graph G:



Although edges in the graph are directed (draw with arrows), for this problem you should **ignore** the arrows and treat G as an **undirected** graph. (Sorry, but the latest version of Microsoft Word prevents me from conveniently editing the graph, which was created in a previous version.)

In the following problems, you should assume that G is represented as a collection of adjacency lists, and that vertices are ordered alphabetically within each adjacency list.

a [10] Use Prim's algorithm to derive a minimum spanning tree of G.

b [10] Use Kruskal's algorithm to derive a minimum spanning tree of G.

c [10] Draw the tree that is induced by performing breadth-first search starting at node d.

d [20] Use Dijkstra's algorithm to determine the shortest paths from d to each of the other vertices. Draw the shortest path tree that is induced by running Dijkstra's algorithm, and annotate each vertex by its shortest path distance from d.

Problem 2 [10] (CLR 23.2-5, p. 476)

Problem 3 [15] (CLR 23.2-6, p. 476) (See CLR p. 89 for a definition of a bipartite graph, and CLR p. 601 for a picture of one.)

Problem 4 [15] (CLR 24.2-6, p. 510)

Problem 5 [10] (A modified version of CLR 25.2-2, p. 531)

Dijkstra's algorithm assumes that the edge weights of the given graph are non-negative. Here we consider some of the consequences of allowing negative edge weights.

a [5] A **negative-weight cycle** of a weighted graph is a cyclic path whose path weight is negative. Suppose that G is a directed graph with a negative-weight cycle. Explain why the single-source shortest path problem may not be well-defined on such a graph.

b [5] If a graph G has some edges with negative weights but no negative-weight cycles, then the single-source shortest path problem is still well-defined. However, Dijkstra's algorithm is not guaranteed to correctly solve the problem in the presence of negative edge weights. Construct a connected, directed, weighted graph with four vertices such that Dijkstra's algorithm gives an incorrect solution to the single-source shortest path problem.

Extra Credit Problem [20] CLR 24.2-8 (p. 510)

Problem Set Header Page
Please make this the first page of your hardcopy submission.

CS231 Problem Set 8
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Name:

Date & Time Submitted *(only if late)*:

Collaborators (*anyone you collaborated with in the process of doing the problem set)*:

*In the **Time** column, please estimate the time you spent on the parts of this problem set. Please try to be as accurate as possible; this information will help me to design future problem sets. I will fill out the **Score** column when grading your problem set.*

Part	Time	Score
General Reading		
Problem 1 [50]		
Problem 2 [10]		
Problem 3 [15]		
Problem 4 [15]		
Problem 5 [10]		
Extra Credit [20]		
Total		