

Assignment 1
Computer Science 235

Reading. Chapter 0 pages 1-28, Section 1.1 pages 31-47

1) Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

- a) $\{1, 3, 5, 7, \dots\}$
- b) $\{w \mid w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w\}$.
- c) $\{n \mid n \text{ is an integer and } n = n + 1\}$

2) Write a formal description of the following sets.

- a) The set containing all integers that are greater than 5.
- b) The set containing the empty string.
- c) The set containing nothing at all.

3) Let A be the set $\{x, y, z\}$, and B be the set $\{x, y\}$.

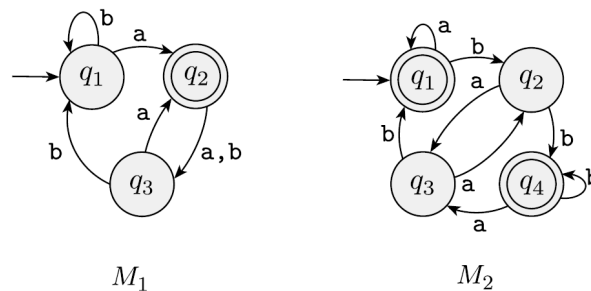
- a) Is A a subset of B ?
- b) Is B a subset of A ?
- c) What is $A \cup B$?
- d) What is $A \cap B$?
- e) What is $A \times B$?
- f) What is the power set of B ?

4) If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

- 5) Let L be the language over the alphabet $\{a,b\}$ generated by the following recursive definition:
Basis. The empty string ϵ belongs to L .
Recursive Step. If w is in L , then so is $aawb$.
Closure. Any string w in L is either ϵ or else it can be obtained by the above recursive definition.

- a) Give a formal symbolic description of the set of strings L encompassed by the above definition. For example, you could state that $L = \{a^n b^3 \mid n = 1, 2, \dots\}$, if this were an accurate description (which it is not).
- b) Using the recursive definition of L directly, prove by induction that if w is in L then a appears twice as many times as b in w .

- 6) The following are the state diagrams of two DFAs, M_1 and M_2 .



Answer the following questions about these machines.

- a) What sequence of states does M_1 go through on input $aabb$?
- b) Does M_1 accept the string $aabb$?
- c) Does M_2 accept the string ϵ ?
- 7) Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is $\{0,1\}$.
- a) $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$.
- b) $\{w \mid w \text{ contains at least three 1s}\}$.
- c) $\{w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length}\}$.
- d) $\{w \mid w \text{ doesn't contain the substring 110}\}$.

- e) $\{w \mid \text{every odd position of } w \text{ is a } 1\}$.
- f) $\{w \mid w \text{ contains at least two } 0\text{s and at most one } 1\}$.
- g) $\{\epsilon, 0\}$.
- h) The empty set.
- i) All strings except the empty string.