

Assignment 5
Computer Science 235

Reading. Sections 2.1, 2.2, and 2.3

1) Consider the following CFG:

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \cdot F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

Give parse trees and derivations for each string.

- a) a
- b) a + a + a

2) Answer each part for the following context-free grammar G .

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \epsilon \\ X &\rightarrow a \mid b \end{aligned}$$

- a) True or False: $T \Rightarrow aba$.
- b) True or False: $T \stackrel{*}{\Rightarrow} aba$.
- c) True or False: $T \Rightarrow T$.
- d) True or False: $XXX \stackrel{*}{\Rightarrow} aba$.
- e) True or False: $S \stackrel{*}{\Rightarrow} \epsilon$.
- f) Give a description in English of $L(G)$.

3) Give context-free grammars that generate the following languages. Assume the alphabet Σ is $\{0, 1\}$.

- a) $\{w \mid \text{the length of } w \text{ is odd}\}$
- b) $\{w \mid w \text{ contains more 1s than 0s}\}$

4) Give state diagrams of pushdown automata for the languages from problem (3) above.

- a) $\{w \mid \text{the length of } w \text{ is odd}\}$
- b) $\{w \mid w \text{ contains more 1s than 0s}\}$

5) Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

6) Let CFG G be the following grammar.

$$\begin{aligned} S &\rightarrow aSb \mid bY \mid Ya \\ Y &\rightarrow bY \mid aY \mid \epsilon \end{aligned}$$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.

7) Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context free and B is regular, then A/B is context free.

8) Show that if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n-1$ steps are required for any derivation of w .