Pushdown Automata

Sipser: Section 2.2 pages 111 - 116

Balanced Brackets

The grammar \( G = (V, \Sigma, R, S) \), where
\[ V = \{ S \}, \]
\[ \Sigma = \{ [, ] \}, \]
\[ R = \{ S \rightarrow \varepsilon \mid SS \mid [S] \} \]
generates all strings of balanced brackets.

Is the language \( L(G) \) regular? Why / Why not?

Recognizing Context-Free Languages

Grammars are language generators. It is not immediately clear how they might be used as language recognizers.

The language \( L(G) \) of balanced brackets is not regular. It cannot be recognized by a finite state automaton.

However, it is very similar to the BEGIN/END blocks of many procedural languages and, therefore, must be recognized by some compiler or interpreter.
Auxiliary Store

We could recognize the language $L(G)$ of balanced brackets by reading left to right, if we could remember left brackets along the way.

[ ] [ ] [ ] [ ]

matches some previous left bracket

Pushdown Automaton

The last left bracket seen matches the first right bracket. This suggests a stack storage mechanism.

Describing a Pushdown Machine

A pushdown automaton is a sextuple $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where

- $Q$ is a finite set of states,
- $\Sigma$ is a finite alphabet (the input symbols),
- $\Gamma$ is a finite alphabet (the stack symbols),
- $\delta: (Q \times \Sigma \times \Gamma_0) \rightarrow P(Q \times \Gamma_0)$ is the transition function,
- $q_0 \in Q$ is the initial state, and
- $F \subseteq Q$ is the set of accept states.
Balanced Brackets

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where

- $Q = \{q_1, q_2, q_3\}$,
- $\Sigma = \{[, ]\}$,
- $\Gamma = \{[, $\}$,
- $q_0 = q_1$,
- $F = \{q_1, q_3\}$, and
- $\delta$ is given by the transition diagram:

Finite Automata and Pushdown Automata

Regular Languages $\Rightarrow$ Pushdown Accept

Proposition. Every finite automaton can be viewed as a pushdown automaton that never operates on its stack.

Proof. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton. Define $M^* = (Q, \Sigma, \Gamma, \delta', q_0, F)$, where …

Pushdown Automata are Nondeterministic

Build a machine to recognize $L(G) = \{ww^R \mid w \in \{0,1\}^*\}$
Pushdown Automata are Nondeterministic

Build a machine to recognize

$L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k \}$