Turing Machines

Sipser: Section 3.1 pages 165 - 175

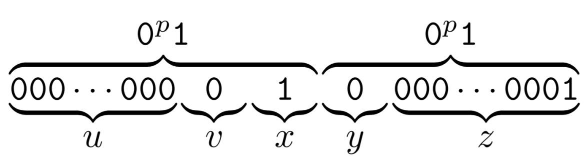
A Context-Free Grammar for $\{a^n b^n c^n : n \geq 0\}$?

Theorem. If A is a context-free language, then there is a number p where, if s is any string in A of length $\geq p$, then $s = uvxyz$ such that:

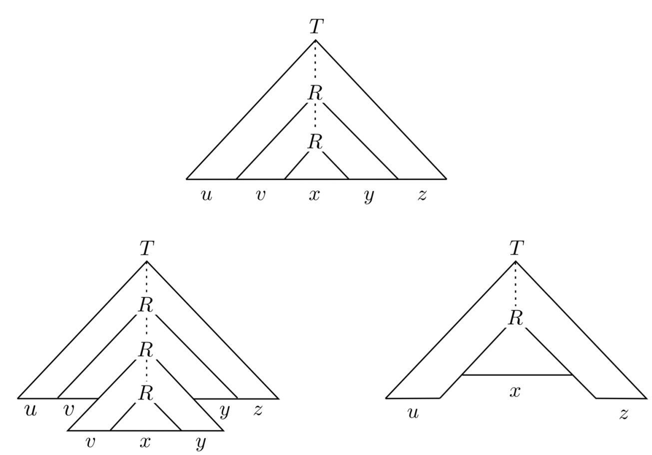
1. For each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

A Context-Free Grammar for $D = \{ww : w \in \{0,1\}^*\}$?

Assume D is a context-free language and reach a contradiction.
Let p be the pumping length given by the pumping lemma.
Choose $s = 0^p10^p1$?



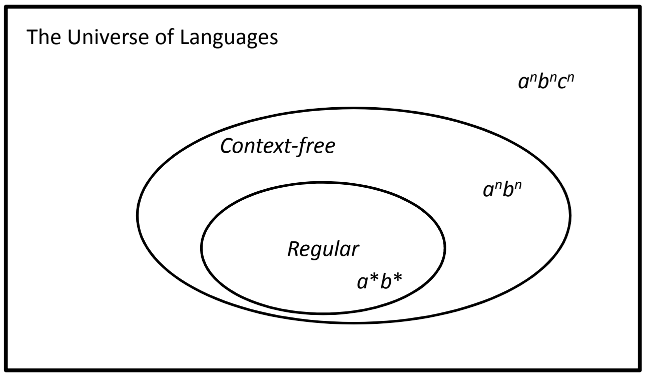
Surgery on Parse Trees



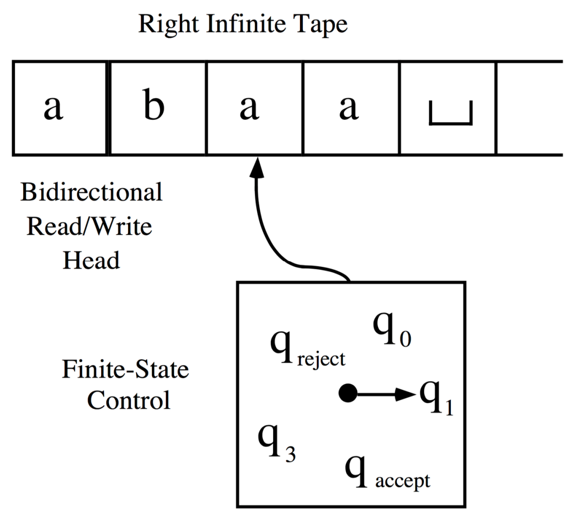
Observation. A k-ary tree of height n has at most k^n leaves.

Chomsky Strikes Again

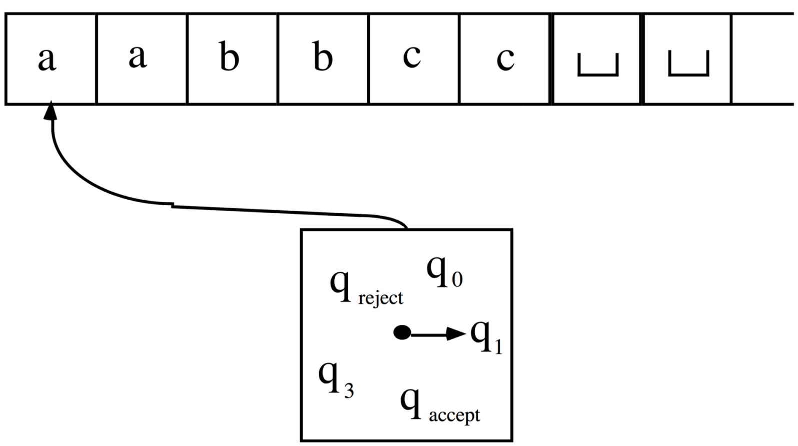
Corollary. Every regular language is context-free.



Turing Machines



Recognizing $\{a^n b^n c^n : n \geq 0\}$

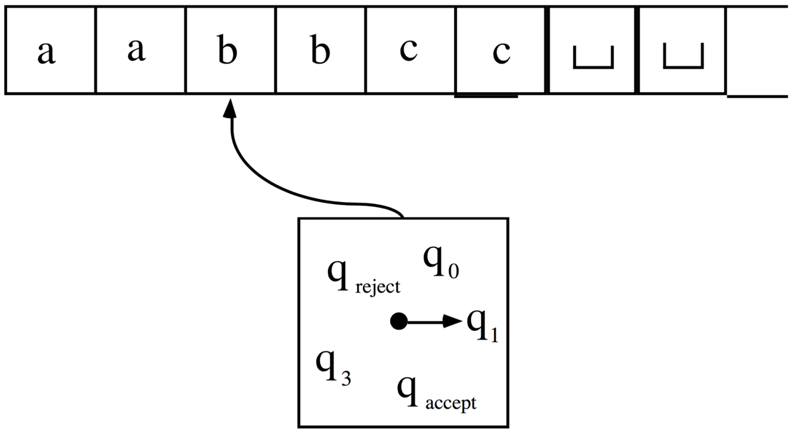


Boring ...

Definition. A ***Turing Machine*** is a 7-tuple, $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, Q, Σ, Γ are finite sets,

1. Q is the set of states,
2. Σ is the input alphabet not containing the special ***blank*** symbol ⊔,
3. Γ is the tape alphabet, where $\{⊔\} \in \Gamma$ and $\Sigma \subseteq \Gamma$,
4. δ: $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function,
5. $q_0 \in Q$ is the start state,
6. $q_{accept} \in Q$ is the accept state, and
7. $q_{reject} \in Q$ is the reject state, where $q_{reject} \neq q_{accept}$.

Configurations and Yields



aaq_1bbcc ***yields*** $aaxq_3bcc$

Recursively Enumerable Languages

Definition. A Turing machine M accepts input w if a sequence of configurations C_1, C_2, …, C_k exists where

1. C_1 is the start configuration of M on input w,
2. each C_i yields C_{i+1}, and
3. C_k is an accepting configuration.

Definition. Call a language ***Turing-recognizable*** if it is the language accepted by some Turing machine.

Recognizing $A = \{0^{2^n} \mid n \geq 0\}$

Define $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where

$Q = \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\}$,

$\Sigma = \{0\}$,

$\Gamma = \{0, x, \sqcup\}$, and

$\delta = Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is given by