What is an Algorithm?

**Quiz**

- Can a TM ever write the blank symbol $\sqcup$ on its tape?
- Can the tape alphabet $\Gamma$ of a TM be the same as the input alphabet $\Sigma$?
- Can a TM’s head ever be in the same location in two successive steps?
- Can a TM contain just a single state?

Give the sequence of configurations that the TM on the right enters when started on the input string $00$.

Computability

**Hilbert’s Tenth Problem.**

Find a process according to which it can be determined by a finite number of operations whether a given polynomial

$$p(x_1, x_2, \ldots, x_n)$$

has an integral root.

Recipes and Processes

**Definition.** An algorithm is a finite sequence of operations, each chosen from a finite set of well-defined operations, that halts in a finite time.
**Church-Turing Thesis**

\[
\text{Algorithm} = \text{Turing Machine}
\]

**Languages and Problems**

**Definition.** Let \( D = \{ p \mid p \text{ is a polynomial with an integral root} \} \).

Hilbert's Tenth Problem.
Determine whether \( D \) is Turing-decidable.

\[
D = \{ p \mid p \text{ is a polynomial with integral roots} \}
\]

\( M = \) "The input is a polynomial \( p(x_1, x_2, \ldots, x_n) \).

1. Lexicographically generate integer values for \((x_1, x_2, \ldots, x_n)\).

2. Evaluate \( p \) as each set of values is generated. If at any point the polynomial evaluates to 0, accept."

**Here We Go Again**

The Universe of Languages

<table>
<thead>
<tr>
<th>Turing-recognizable</th>
<th>Turing-decidable</th>
<th>Context-free</th>
<th>Regular</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^*b^<em>c^</em> )</td>
<td>( a^<em>b^</em> )</td>
<td>( a^<em>b^</em> )</td>
<td></td>
</tr>
</tbody>
</table>
Describing Turing Machines

- **Formal**

- **Implementation**
  \[ M = \text{"On input string } w:\]
  1. Sweep across tape, crossing off every other 0.
  2. If tape contained one 0, accept.
  3. Else, if number of 0's is odd, reject.
  4. Return head to left-hand end of tape.
  5. Go to step 1.

- **High-level**
  
  repeat until \( n = 1 \)
  exit if \( n \mod 2 \neq 0 \)
  set \( n = n \div 2 \)

Exercises

- Is a TM with a doubly infinite tape (infinite to the left as well as the right) more powerful than an ordinary TM?

- Let a \( k \)-PDA be a pushdown automaton that has \( k \) stacks. Is a 1-PDA more powerful than a 0-PDA? Is a 2-PDA more powerful than a 1-PDA? Is a 3-PDA more powerful than a 2-PDA?