What Machines Can Do

Sipser: Section 4.1 pages 193 - 201

Problems Concerning Regular Languages: Acceptance

Definition.

A_{DFA} = {<*B*, *w*> | *B* is a DFA that accepts input string *w* }

A_{NFA} = {<*B*, *w*> | *B* is an NFA that accepts input string *w* }

A_{REX} = {<*R*, *w*> | *R* is a regular expression that generates string *w* }

Deciding Regular Languages

Theorem. A_{DFA} is decidable.

Proof. *M* = "On input <*B*, *w*>,

1. Simulate *B* on input *w*.
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*."

Guessing is No Problem

Theorem. A_{NFA} is decidable.

Proof. *N* = "On input <*B*, *w*>,

1. Convert NFA *B* to an equivalent DFA *C*.
2. Simulate TM *M* on input <*C*, *w*>.
3. If *M* accepts, *accept*. Otherwise, *reject*."

Deciding Regular Expressions

Theorem. A_{REX} is decidable.

Proof. P = "On input $\langle R, w \rangle$,

1. Convert RE R to an equivalent DFA C.
2. Simulate TM M on input $\langle C, w \rangle$.
3. If M accepts, *accept*. Otherwise, *reject*."

Emptiness Testing

Definition. $E_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \varnothing\}$

Is E_{DFA} decidable?

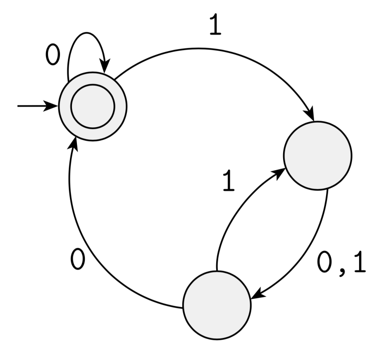
Equivalence Testing

Definition. $EQ_{DFA} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$

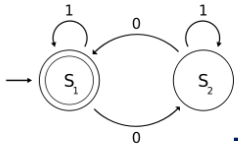
Is EQ_{DFA} decidable?

Exercises

1. For the DFA M on the right
   a. Is $\langle M, 0100 \rangle \in A_{DFA}$?
   b. Is $\langle M, 010 \rangle \in A_{DFA}$?
   c. Is $\langle M \rangle \in A_{DFA}$?
   d. Is $\langle M, 0100 \rangle \in A_{REX}$?
   e. Is $\langle M \rangle \in E_{DFA}$?
   f. Is $\langle M, M \rangle \in EQ_{DFA}$?

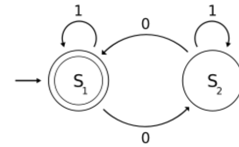


2. Let $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$.
   Show that ALL_{DFA} is decidable.



Deciding Context-Free Languages?

Definition. A_{CFG} = {<*G*, *w*> | *G* is a CFG that generates *w* }



Yes! (What a Relief)

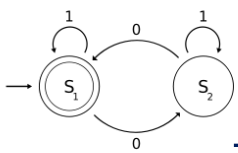
Theorem. A_{CFG} is decidable.

Proof. *S* = "On input <*G*, *w*>,

1. Convert *G* to an equivalent grammar in Chomsky normal form.

2. List all derivations with 2*n*-1 steps, where *n* is the length of *w*.

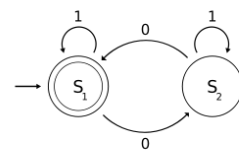
3. If any of these derivations generate *w*, *accept*. Otherwise, *reject*."



Emptiness Testing

Definition. E_{CFG} = {<*G*> | *G* is a CFG and *L*(*G*) = ∅ }

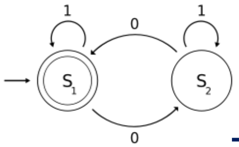
Is E_{CFG} decidable?



Equivalence Testing

Definition. EQ_{CFG} = {<*G*, *H* > | *G* and *H* are CFGs and *L*(*G*) = *L*(*H*) }

Is EQ_{CFG} decidable?



The Missing Piece

Theorem. Every context-free language is decidable.

Proof. Let G be a CFG for A. We design a TM M_G that decides A as follows.

M_G = "On input w.

1. Run TM S on input <G, w>.

2. If this machine accepts, *accept*. Otherwise, *reject*."