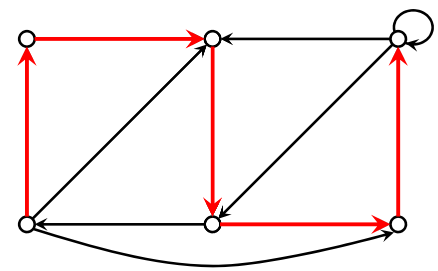
NP-Complete Problems

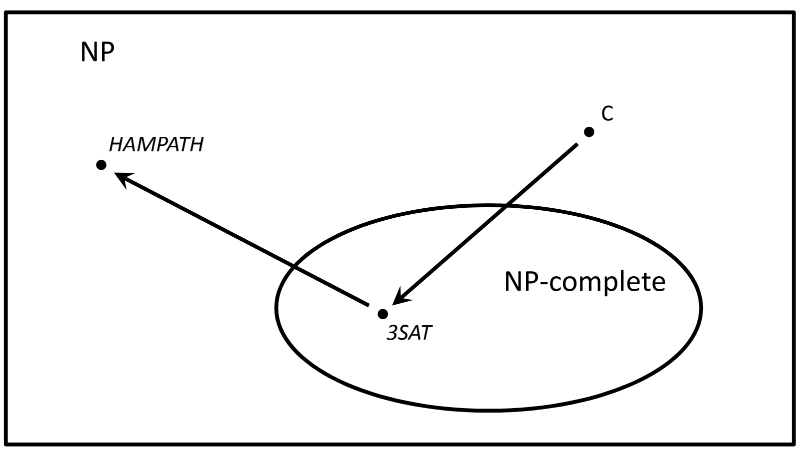
Sipser: Section 7.5 pages 311 - 322

More NP-Complete Problems

Theorem. HAMILTONIAN PATH is NP-complete.



We Reduce *3SAT* to *HAMPATH*



More Precisely,

Proof Outline. Given an instance of *3SAT*,

$$\varphi = (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (x_2 \vee \neg x_4 \vee x_5)$$

we construct a directed graph *G* so that φ is satisfiable iff *G* has a Hamiltonian path.

3SAT's Salient Features*

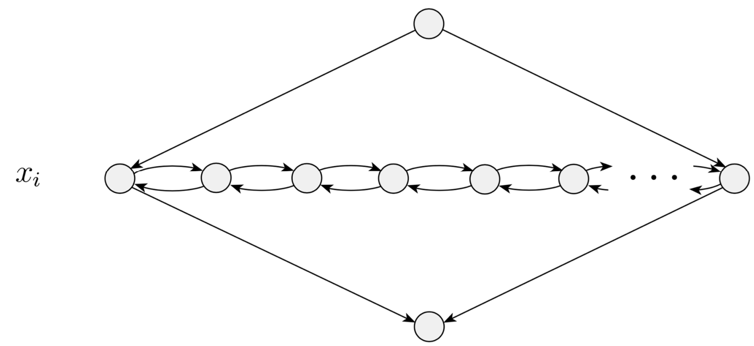
Choice. Each variable has a choice between two truth values.

Consistency. Different occurrences of the same variable have the same value.

Constraints. Variable occurrences are organized into *clauses* that provide constraints that must be satisfied.

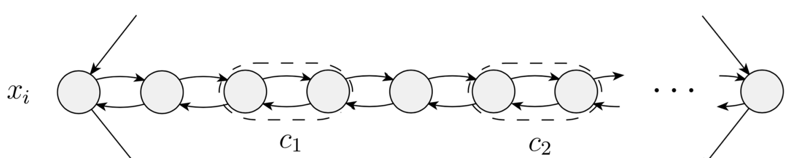
* We model each of these three features by a different "gadget" in the graph *G*.

Modeling Variable x_i*

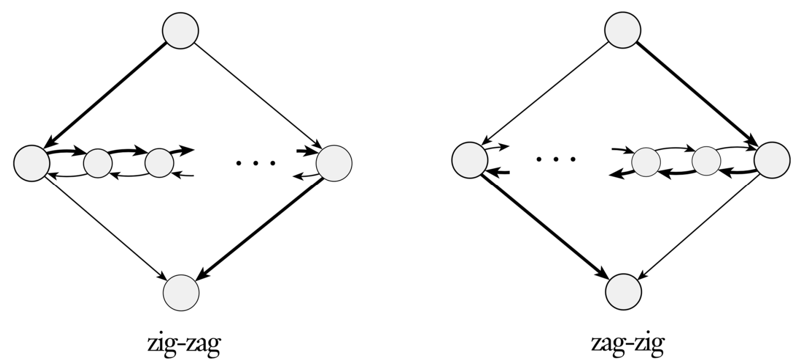


* The choice gadget.

The Consistency Gadget



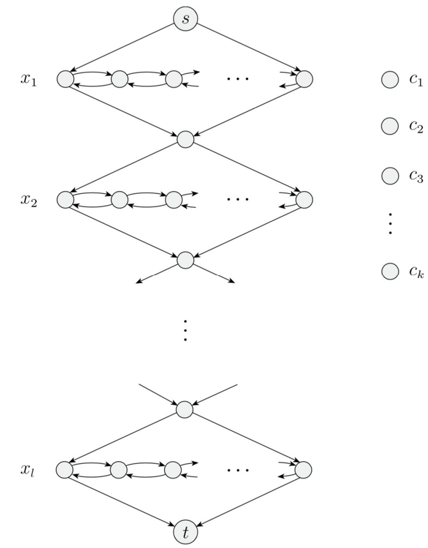
Zig-zagging and Zag-zigging



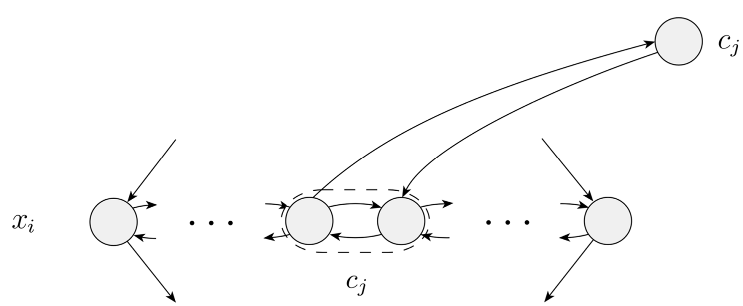
* The consistency gadget in action.

Modeling Clause c_j

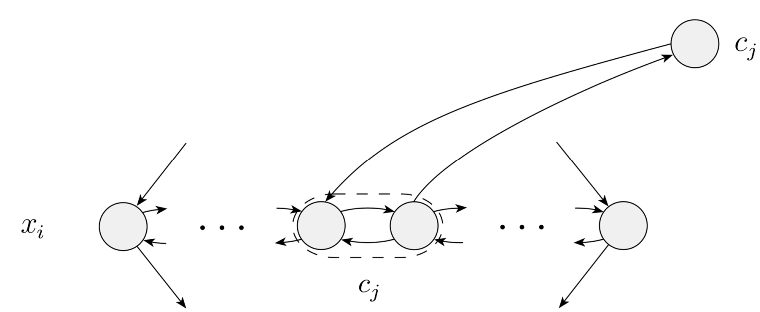
The Big Picture



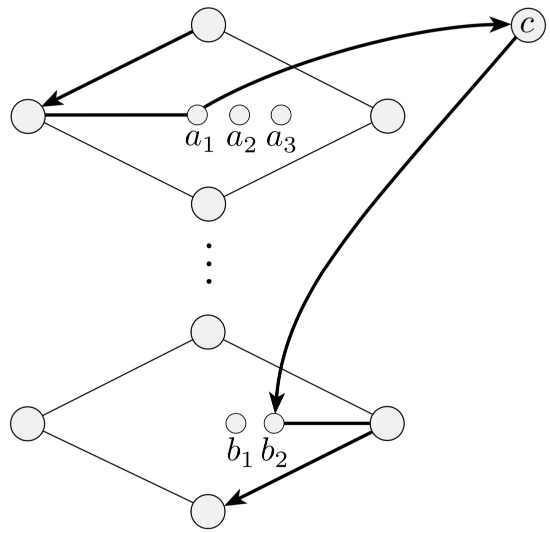
The Constraint Gadget when c_j Contains x_i



The Constraint Gadget when c_j Contains $\neg x_i$



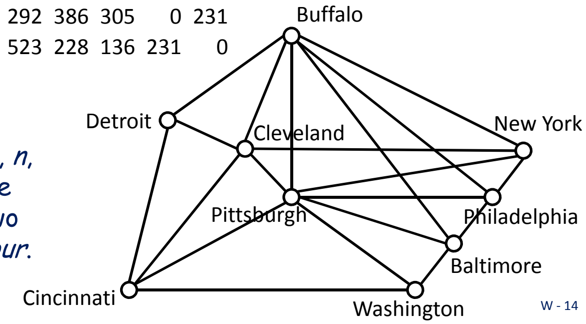
A Situation that Cannot Occur



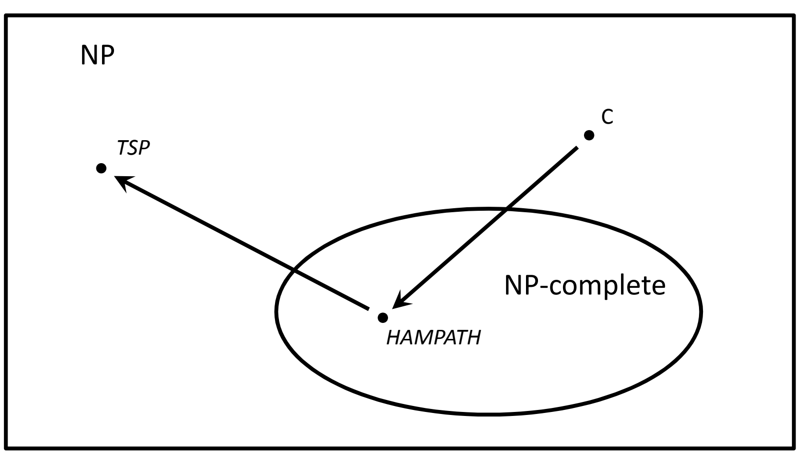
TSP is NP-Complete

Baltimore	0	345	514	355	522	189	97	230	39
Buffalo	345	0	430	186	252	445	365	217	384
Cincinnati	514	430	0	244	265	670	589	284	492
Cleveland	355	186	244	0	167	507	430	125	356
Detroit	522	252	265	167	0	674	597	292	523
New York	189	445	670	507	674	0	92	386	228
Philadelphia	97	365	589	430	597	92	0	305	136
Pittsburgh	230	217	284	125	292	386	305	0	231
Washington	39	384	492	356	523	228	136	231	0

TSP: Given *n* cities, 1, 2, …, *n*, together with a nonnegative distance d_{ij} between any two cities, find the *shortest tour*.



We Reduce *HAMPATH* to *TSP*



SUBSET-SUM is NP-Complete

SUBSET-SUM: Given a set of integers, does any subset sum to *t*?

45 -18 4 16 -21 115
201 -8 -12
-64 -17 14 61 94
-190 -89 51 -79
77 23 -106
48 57 -35
141 -219 28 81

SUBSET-SUM is NP-Complete

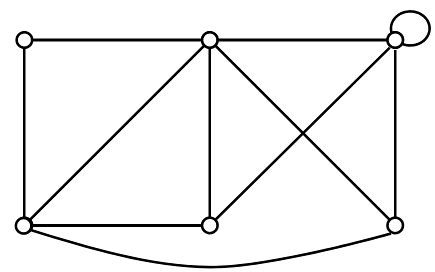
We show *3SAT* $\leq_p$ *SUBSET-SUM* as in the following example.
Given
$(x_1 \vee \neg x_2 \vee x_3) \wedge$
$(x_2 \vee x_3 \vee \ldots) \wedge$
$\ldots \wedge$
$(\neg x_3 \vee \ldots)$
we construct:

	1	2	3	4	$\cdots$	l	c_1	c_2	$\cdots$	c_k
y_1	1	0	0	0	$\cdots$	0	1	0	$\cdots$	0
z_1	1	0	0	0	$\cdots$	0	0	0	$\cdots$	0
y_2		1	0	0	$\cdots$	0	0	1	$\cdots$	0
z_2		1	0	0	$\cdots$	0	1	0	$\cdots$	0
y_3			1	0	$\cdots$	0	1	1	$\cdots$	0
z_3			1	0	$\cdots$	0	0	0	$\cdots$	1
$\vdots$					$\ddots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
y_l						1	0	0	$\cdots$	0
z_l						1	0	0	$\cdots$	0
g_1							1	0	$\cdots$	0
h_1							1	0	$\cdots$	0
g_2								1	$\cdots$	0
h_2								1	$\cdots$	0
$\vdots$									$\ddots$	$\vdots$
g_k										1
h_k										1
t	1	1	1	1	$\cdots$	1	3	3	$\cdots$	3

VERTEX-COVER is NP-Complete

If *G* is an undirected graph, a ***vertex cover*** of *G* is a subset of the nodes where every edge of *G* touches one of those nodes.

Theorem. *VERTEX-COVER* is NP-Complete.



SET-COVER is NP-Complete

Given a set *S* and a collection of subsets from *S*, do any *k* of the subsets unioned together equal *S*?

***S* = {red, blue, yellow, green, purple, brown, silver, black, gold, white, orange, pink}**

Subsets:

{red, blue, silver, pink}
{yellow, gold, white, orange}
{green, silver, gold, white}
{red, green, black, orange}
{green, purple, brown, black}
{brown, black, white, pink}

Theorem. *SET-COVER* is NP-Complete.

SET-COVER is NP-Complete

Theorem. *SET-COVER* is NP-Complete.

***S* = {A, B, C, D, E, F, G, H, I, J, K}**

Subsets:

{A, D}
{B, C, F, H}
{E, H, J}
{A, B, E, G, I}
{F, I, K}
{D, G, J, K}