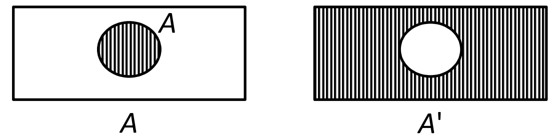
Language Recognizers

Sipser: Section 1.1 pages 40 - 47

Building New Languages From Old

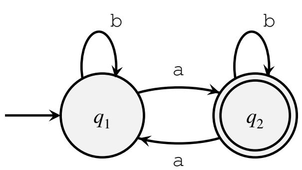
Let A and B be languages. We define the **complement** of A as $A' = \{x \mid x \notin A\}$.



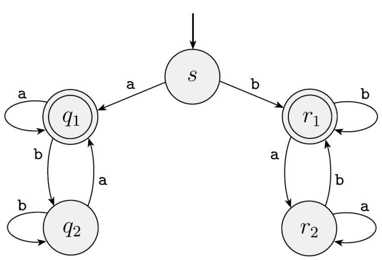
Let A = { w | w is a string of 0s and 1s containing an odd number of 1s }. What is A'?

Let B = { w | w is a string over {a,b} that starts and ends with the same symbol }. What is B'?

Which Complements are Regular?



Let A = { w | w is a string of as and bs containing an odd number of as }. Is A' regular?



Let B = { w | w is a string over {a,b} that starts and ends with the same symbol }. Is B' regular?

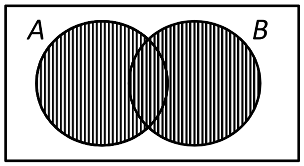
Our First Theorem and Proof

Theorem. The class of regular languages is closed under the complement operation.

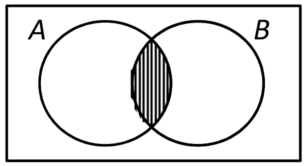
Adding and Subtracting Languages

Let A and B be languages. We define the **union** of A and B as $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

We define the **intersection** of A and B as $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.



$A \cup B$



$A \cap B$

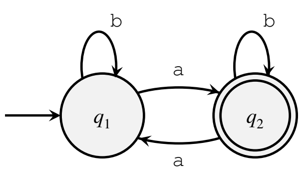
For Example

Let A = { w | w is a string of as and bs containing an odd number of as }, and let B = { w | w is a string of as and bs that starts and ends with the same symbol }.

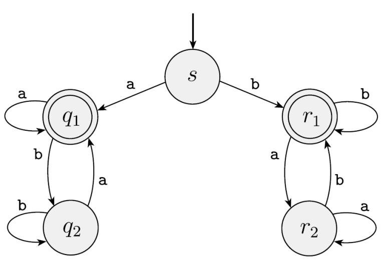
What is $A \cap B$?

Is $A \cap B$ regular?

Build a Machine



Language A = { w | w is a string of as and bs containing an odd number of as } is recognized by the machine M_1.



Language B = { w | w is a string over $\{a,b\}$ that starts and ends with the same symbol } is recognized by the machine M_2.

Our Second Theorem and Proof

Theorem. The class of regular languages is closed under the intersection operation.

Another Theorem

Theorem. The class of regular languages is closed under the union operation.

Concatenation

Let *A* and *B* be languages. We define the concatenation of *A* and *B*, $A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$.

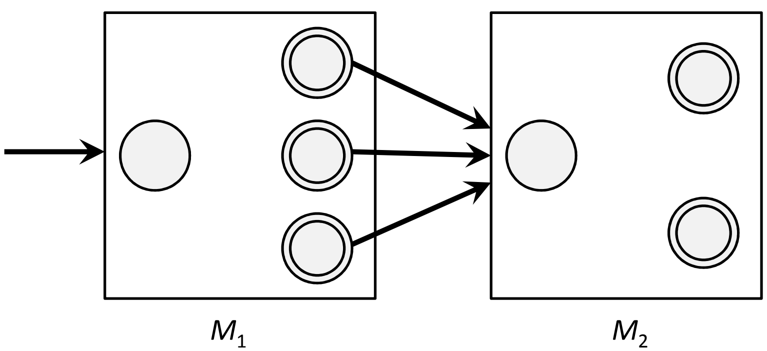
Let $A = \{ w \mid w$ is a string of 0s and 1s containing an odd number of 1s $\}$ and let $B = \{ w \mid w$ is a string of 0s and 1s containing an even number of 1s $\}$.

What are $A \circ B$; $B \circ A$; $A \circ A$; $B \circ B$? Are any of these languages regular?

Closure under Concatenation

Conjecture. The class of regular languages is closed under concatenation.

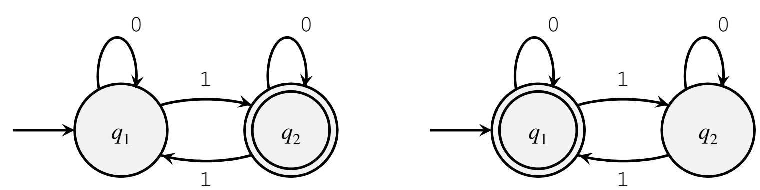
Proof idea.



Checking Out Our Approach

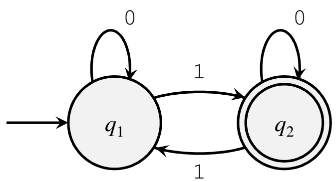
Let $A = \{ w \mid w$ is a string of 0s and 1s containing an odd number of 1s $\}$, and let $B = \{ w \mid w$ is a string of 0s and 1s containing an even number of 1s $\}$.

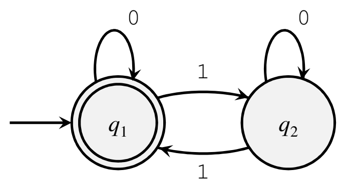
We construct machines M_1 and M_2 that recognize languages *A* and *B*, respectively, then follow proof idea to glue them together.



Coming Attractions: Nondeterminism

We need to relax the hard and fast rules defining a finite state machine.





Kleene Star

Let A be a language. We define $A^* = \{ x_1x_2\ldots x_k \mid k \geq 0$ and each $x_i \in A \}$.

Let $A = \{0, 1\}$, let $B = \{ w \mid w$ is a string of 0s and 1s containing an even number of 1s $\}$, and let $C = \{ w \mid w$ is a string of 0s and 1s containing an odd number of 1s $\}$.

What are A^*, B^*, and C^*?