Building New Languages From Old

Let $A$ and $B$ be languages. We define the complement of $A$ as $A' = \{ x \mid x \notin A \}$.

Let $A = \{ w \mid w$ is a string of 0s and 1s containing an odd number of 1s $\}$. What is $A'$?

Let $B = \{ w \mid w$ is a string over $\{a,b\}$ that starts and ends with the same symbol $\}$. What is $B'$?

Our First Theorem and Proof

Theorem. The class of regular languages is closed under the complement operation.

Which Complements are Regular?

Let $A = \{ w \mid w$ is a string of $a$s and $b$s containing an odd number of $a$s $\}$. Is $A'$ regular?

Let $B = \{ w \mid w$ is a string over $\{a,b\}$ that starts and ends with the same symbol $\}$. Is $B'$ regular?
Adding and Subtracting Languages

Let $A$ and $B$ be languages. We define the union of $A$ and $B$ as $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$.

We define the intersection of $A$ and $B$ as $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$.

For Example

Let $A = \{ w \mid w \text{ is a string of } a \text{ and } b \text{ containing an odd number of } a \}$, and let $B = \{ w \mid w \text{ is a string of } a \text{ and } b \text{ that starts and ends with the same symbol} \}$.

What is $A \cap B$?

Is $A \cap B$ regular?

Build a Machine

Language $A = \{ w \mid w \text{ is a string of } a \text{ and } b \text{ containing an odd number of } a \}$ is recognized by the machine $M_1$.

Language $B = \{ w \mid w \text{ is a string over } \{a,b\} \text{ that starts and ends with the same symbol} \}$ is recognized by the machine $M_2$.

Our Second Theorem and Proof

Theorem. The class of regular languages is closed under the intersection operation.
Another Theorem

**Theorem.** The class of regular languages is closed under the union operation.

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**Concatenation**

Let $A$ and $B$ be languages. We define the concatenation of $A$ and $B$, $A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$. Let $A = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s} \}$ and let $B = \{ w \mid w \text{ is a string of 0s and 1s containing an even number of 1s} \}$. What are $A \circ B$; $B \circ A$; $A \circ A$; $B \circ B$? Are any of these languages regular?

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**Closure under Concatenation**

**Conjecture.** The class of regular languages is closed under concatenation.

**Proof idea.**

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**Checking Out Our Approach**

Let $A = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s} \}$, and let $B = \{ w \mid w \text{ is a string of 0s and 1s containing an even number of 1s} \}$. We construct machines $M_1$ and $M_2$ that recognize languages $A$ and $B$, respectively, then follow proof idea to glue them together.
We need to relax the hard and fast rules defining a finite state machine.

Let $A$ be a language. We define $A^* = \{ x_1x_2...x_k \mid k \geq 0$ and each $x_i \in A \}$.

Let $A = \{0, 1\}$, let $B = \{ w \mid w$ is a string of 0s and 1s containing an even number of 1s $\}$, and let $C = \{ w \mid w$ is a string of 0s and 1s containing an odd number of 1s $\}$.

What are $A^*$, $B^*$, and $C^*$?