Review
Administrivia

- Syllabus and schedule on website: [http://cs.wellesley.edu/~cs235/](http://cs.wellesley.edu/~cs235/)
  - Will post slides, handouts on it
  - Make sure you are subscribed to the Google group: **CS235-01-FA19**

- Textbook
  - Sipser, Introduction to the Theory of Computation, 3rd edition
Administrivia

- Assignment distribution and submission through Gradescope
  - Sign up using the course code: (97KPXY)
  - Due at 9pm

- Assignments need to be typeset, preferably using LaTeX
  - LaTeX resources posted on course site; template will be provided

- Late policy: four (4) late passes
  - Have to communicate via email by the due date
  - Weekends and holidays do not count as late days
Review of Concepts
Relations

A binary relation \( R \) on \( S \times S \) is

- **Reflexive** if for any \( a \in S \), \((a, a) \in R\)
- **Symmetric** if for any \((a, b) \in R\) then \((b, a) \in R\)
- **Transitive** if \((a, b) \in R\) and \((b, c) \in R\) then \((a, c) \in R\)

A relation that satisfies all three is an **equivalence relation**

Q. Identify the properties in these relations:
(a) Lives-in-the-same-city-as, (b) Is-an-ancestor-of, and (c) Is-a-Brother-of
Functions

- A function \( f : D \rightarrow R \) is a relation on \( D \times R \) such that
  for all \( x \in D \), there is exactly one \( y \in R \), such that, \( (x, y) \in f \)

- (input, output) pairs: each input must have a unique output!

- \( (x, y) \in f \) is written as \( f(x) = y \)

- Which of these relations are NOT functions?

  \[ R_1 = \{(0, 0), (1, 1), (2, 4), (3, 9), (4, 16)\} \]
  \[ R_2 = \{(\text{red}, 0), (\text{red}, 1), (\text{blue}, 2)\} \]
  \[ R_3 = \{(\text{small, short}), (\text{medium, middle}), (\text{medium, average}), (\text{large, tall})\} \]
Functions

A function $f : D \rightarrow R$ is

- **One-one** if $f(x_1) = f(x_2)$ then $x_1 = x_2$

- **Onto** for every $y \in R$ there is an $x \in S$ such that $f(x) = y$

- **Bijection (one-one and onto)**

**Q. Identify the properties in these functions:**

$f : \mathbb{Z} \rightarrow \mathbb{Z}$ $\quad f(x) = x + 5, f(x) = 2x, f(x) = x^2$
Proofs

A proof is a logical argument showing that a statement is either true or false

- **Proof by construction**

- **Proof by induction**

- **Proof by contradiction**
Proofs

A proof is a logical argument showing that a statement is either true or false

- **Proof by construction**
  - Prove the existence of an object by constructing it

- **Proof by induction**
  - Show all elements of an infinite set have a certain property

- **Proof by contradiction**
  - Assume the claim is false and reach a contradiction
Coming up with Proofs: Some tips

First read the statement carefully and “unpack”

Types of theorem statements

- If $A$, then $B$ \[ A \implies B \]

- $A$ if and only if $B$ \[ A \iff B \]
  - Prove if $A$, then $B$
  - Prove if $B$, then $A$

- For all $x$, $P(x)$ is true
  - Pick an arbitrary $x$ and prove $P(x)$
Coming up with Proofs: Some tips

Common pitfalls

- Confusing *converse* of a statement with its *contrapositive*

- Statement **X**: *If it is raining, then the ground is wet.*

- Converse: *(Not the same as X)* If the ground is wet, then it is raining.

- Contrapositive: *(Same as X)* If the ground is not wet, then it is not raining.
Coming up with Proofs: Some tips

- Be patient
- Start from first principles and build on it
- Review related theorems/definitions that could be helpful
- Sleep on it
Exercise

- Is the relation *Is-a-Brother-of* transitive?

- Prove that for each even number $n$ greater than 2, there exists a 3-regular graph with $n$ nodes.
Languages
Strings

- **Alphabet** is a finite, nonempty set of symbols

  \[ \Sigma_1 = \{a, b, c\}, \Sigma_2 = \{0, 1\} \]

- **String** is a finite sequence of symbols from an alphabet

  \begin{align*}
  &a, \ b, \ abc, \ abbb, \ ccc \\
  &01, \ 000, \ 11010
  \end{align*}

- **Empty string**: a sequence of length zero \( \mathcal{E} \)

- \( \Sigma^* \) : set of all strings over \( \Sigma \)
Strings

- **Length** of a string $w$ is $|w|$
  
  $|ab| = |01| = 2$
  $|aaa| = |abc| = |ccc| = |000| = 3$

- $Q$. What is $|\varepsilon|$?

- **Concatenation** of strings $x . y$

  If $x = abba$ and $y = b$ then $x . y = abbab$

- A string $z$ is a **substring** of $w$ if $z$ appears consecutively within $w$

  E.g. $bba$ and $bbb$ are both substrings of $abbbbaa$
Languages

- **Language** is any set of strings over a given alphabet \( A \subseteq \Sigma^* \)
- Examples of languages
  - Empty set \( \emptyset \)
  - \( \{\varepsilon\} \)
  - The set of all words in The Oxford English Dictionary
  - The set of all binary strings with odd number of 1s
  - The set of all python programs that print “Hello World!”
- A language can be finite or infinite
- Formal language different from languages such as English or Python
  - May not carry any meaning
Exercise

- Need to decide if a certain string belongs to a certain language

- How can this be done?

- How can this be done when…
  - You cannot write anything down nor remember what you saw before
  - You cannot look ahead