Let $A = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s} \}$ and let $B = \{ w \mid w \text{ is a string of 0s and 1s containing an even number of 1s} \}$.

Construct a (nondeterministic) machine $N$ that recognizes language $A \circ B$.
How Does That Compute?

Deterministic computation

Nondeterministic computation

start

\[ \vdots \]

\begin{align*}
&\text{reject} \\
&\vdots \\
&\text{accept or reject}
\end{align*}

For Example

\[ \mathcal{N}_1 \]

Input 010110

Symbol read

\[ \text{Start} \]

\begin{align*}
&0 \\
&1 \\
&0 \\
&1 \\
&0 \\
&1 \\
&0 \\
&1
\end{align*}

Another Example

Formally

A nondeterministic finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where

1. \(Q\) is a finite set called the states,
2. \(\Sigma\) is a finite set called the alphabet,
3. \(\delta: Q \times \Sigma \to 2^Q\) is the transition function,
4. \(q_0 \in Q\) is the start state, and
5. \(F \subseteq Q\) is the set of accept states.
Nondeterministic Automata Computation

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and $w$ a string over the alphabet $\Sigma$. Then $N$ accepts $w$ if we can write $w$ as $w = y_1y_2\ldots y_m$, where each $y_i$ is a member of $\Sigma$ and a sequence of states $r_0, r_1, \ldots, r_m$ exists in $Q$ with three conditions:

1. $r_0 = q_0$.
2. $r_{i+1} \in \delta(r_i, y_{i+1})$, for $i = 0, \ldots, m-1$, and
3. $r_m \in F$.

Nondeterminism is Your Friend

Build an NFA that recognizes the language $B = \{ w \mid w \text{ is a string of } a \text{ and } b \text{ that starts and ends with the same symbol and contains at least two symbols} \}$.

When You Can't Prove What You Want...

Theorem. The class of regular languages is closed under concatenation.

Proof.

Prove What You Can

Theorem. The class of regular languages recognized by NFAs is closed under concatenation.

Proof.
Theorem. The class of regular languages recognized by NFAs is closed under Kleene star.

Proof. ... somebody showed that the class of languages accepted by NFAs and the class of languages accepted by DFAs were equal ...

Theorem. A language is regular if and only if it is accepted by a nondeterministic finite automata.

We Would Have ...

Corollary. The class of regular languages is closed under concatenation.

Corollary. The class of regular languages is closed under Kleene star.

We Also Have a Nifty Proof of Closure Under Unions*

Theorem. The class of regular languages is closed under union.

Proof. ...

*Which turns out to help construct machines for recognizing regular languages.