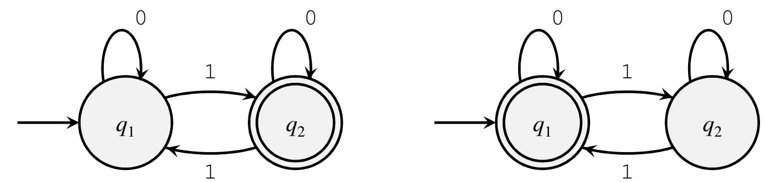
Nondeterminism

Sipser: Section 1.2 pages 47 - 54

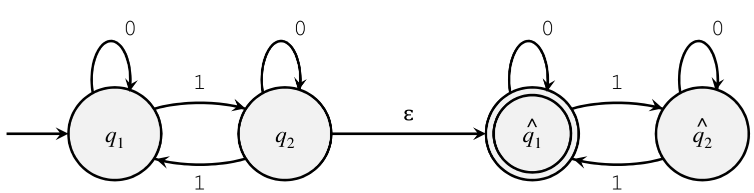
Concatenating Languages and Machines

Let A = { w | w is a string of 0s and 1s containing an odd number of 1s } and let B = { w | w is a string of 0s and 1s containing an even number of 1s }.



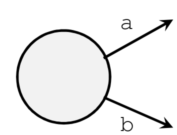
Construct a (nondeterministic) machine N that recognizes language $A \circ B$.

Something's Fishy

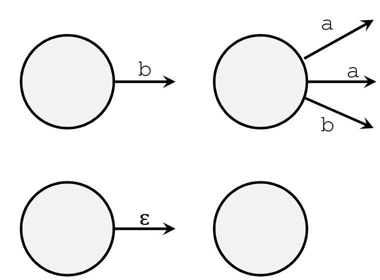


Relaxing the Rules

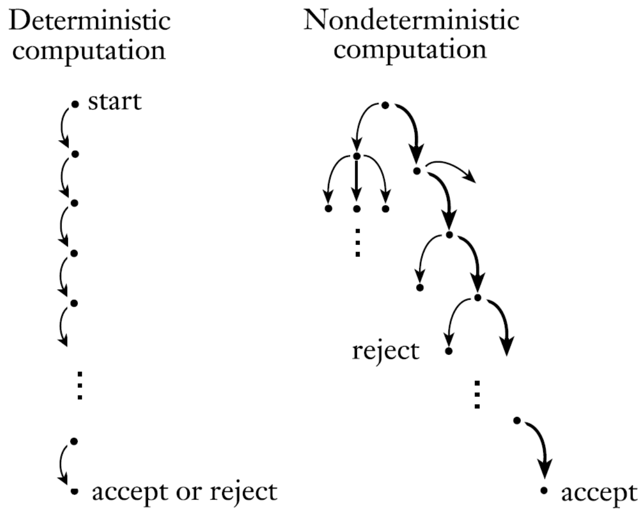
Deterministic (DFA)



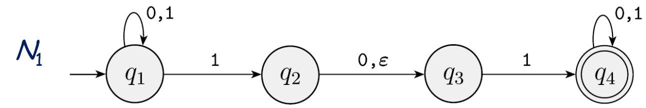
Nondeterministic (NFA)



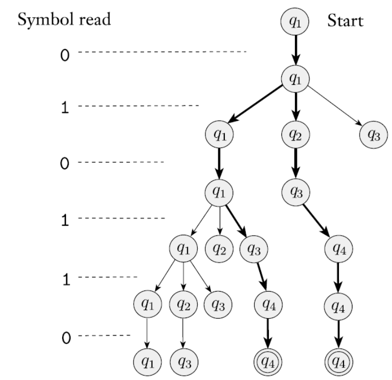
How Does That Compute?



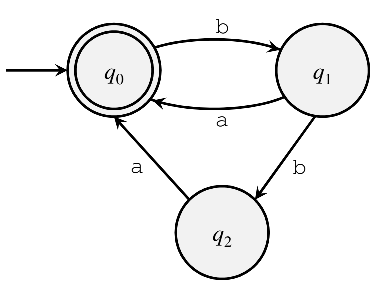
For Example



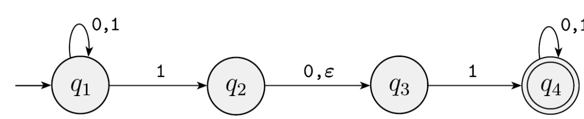
Input 010110



Another Example



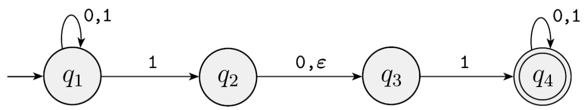
Formally



A **nondeterministic finite automaton** is a 5-tuple (Q, Σ, δ, q_0, F), where

1. Q is a finite set called the **states**,
2. Σ is a finite set called the **alphabet**,
3. δ: $Q \times \Sigma_\varepsilon \rightarrow P(Q)$ is the **transition function**,
4. $q_0 \in Q$ is the **start state**, and
5. $F \subseteq Q$ is the **set of accept states**.

Nondeterministic Automata Computation



Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and w a string over the alphabet Σ. Then N **accepts** w if we can write w as $w = y_1 y_2 \cdots y_m$, where each y_i is a member of Σ_ε and a sequence of states $r_0, r_1, \ldots, r_m$ exists in Q with three conditions:

1. $r_0 = q_0$,
2. $r_{i+1} \in \delta(r_i, y_{i+1})$, for $i = 0, \ldots, m-1$, and
3. $r_m \in F$.

Nondeterminism is Your Friend

Build an NFA that recognizes the language B = { w | w is a string of *a*s and *b*s that starts and ends with the same symbol and contains at least two symbols }.

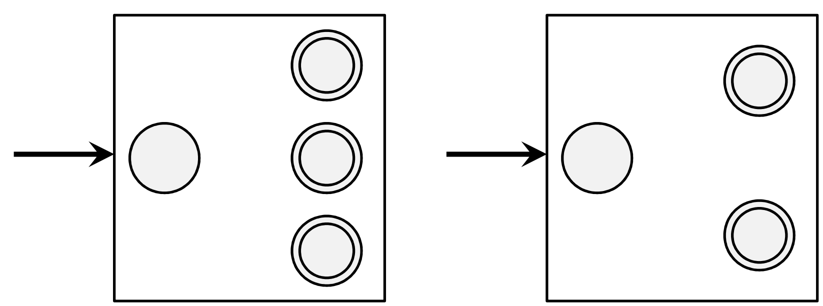
When You Can't Prove What You Want...

Theorem. The class of regular languages is closed under concatenation.

Prove What You Can

Theorem. The class of regular languages recognized by NFAs is closed under concatenation.

Proof.

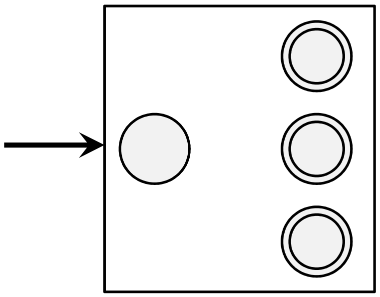


N_1 N_2

Kleene Star

Theorem. The class of regular languages recognized by NFAs is closed under Kleene star.

Proof.



N_1

Suppose ...

... somebody showed that the class of languages accepted by NFAs and the class of languages accepted by DFAs were equal ...

Theorem. A language is regular if and only if it is accepted by a nondeterministic finite automata.

We Would Have ...

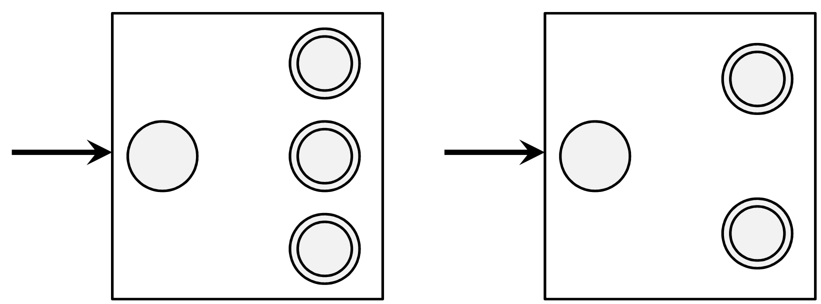
Corollary. The class of regular languages is closed under concatenation.

Corollary. The class of regular languages is closed under Kleene star.

We Also Have a Nifty Proof of Closure Under Unions*

Theorem. The class of regular languages is closed under union.

Proof.



N_1 N_2

*Which turns out to help construct machines for recognizing regular languages.