Closure Under Regular Operations

Warm Up

Design an NFA that recognizes the language 
\( \{ w \in \{a\}^* \mid |w| \text{ is divisible by 3 or 5} \} \).

Closure of Regular Languages Under Union Using NFAs

**Theorem.** The class of regular languages is closed under the union operation.

**Proof.**
Closure of Regular Languages Under Union Using NFAs

**Theorem.** The class of regular languages is closed under the union operation.

**Proof.** Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$, for a new state $q_0$.
2. $q_0$ is the start state of $N$.
3. $F = F_1 \cup F_2$.
4. For any $q \in Q$ and any $a \in \Sigma$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

Closure of Regular Languages Under Concatenation Using NFAs

**Theorem.** The class of regular languages is closed under the concatenation operation.

**Proof.** Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$.

Closure of Regular Languages Under Kleene Star Using NFAs

**Theorem.** The class of regular languages is closed under the Kleene star operation.

**Proof.**
The class of regular languages is closed under the Kleene star operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$. Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1^*$. Prove that every NFA can be converted to an equivalent one that has a single accept state.