Closure Under Regular Operations

Sipser: Section 1.2 pages 58 - 63

Warm Up

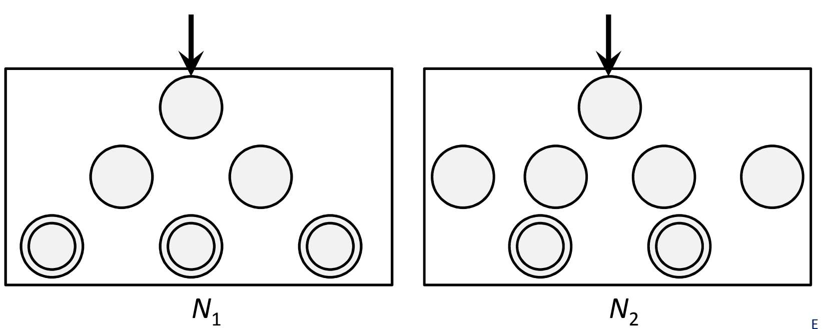
Design an NFA that recognizes the language

$\{ w \in \{a\}^* \mid |w| \text{ is divisible by 3 or 5} \}$.

Closure of Regular Languages Under Union Using NFAs

Theorem. The class of regular languages is closed under the union operation.

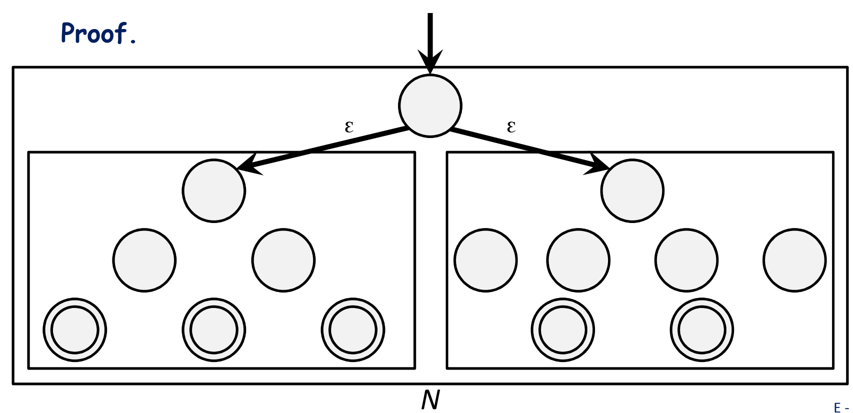
Proof.

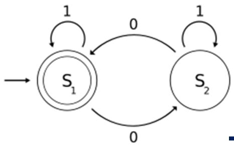


Closure of Regular Languages Under Union Using NFAs

Theorem. The class of regular languages is closed under the union operation.

Proof.





Closure of Regular Languages Under Union Using NFAs

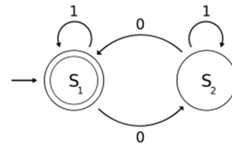
Theorem. The class of regular languages is closed under the union operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1 and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

1. $Q = \{q_0\} \cup Q_1 \cup Q_2$, for a new state q_0.
2. q_0 is the start state of N.
3. $F = F_1 \cup F_2$.
4. For any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

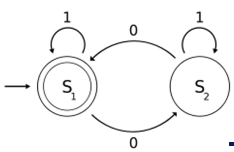
$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$



Closure of Regular Languages Under Concatenation Using NFAs

Theorem. The class of regular languages is closed under the concatenation operation.

Proof.

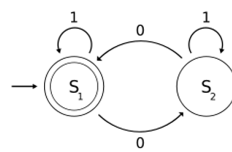


Closure of Regular Languages Under Concatenation Using NFAs

Theorem. The class of regular languages is closed under the concatenation operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1 and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

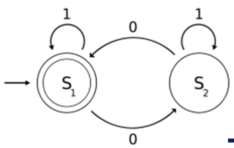
Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$.



Closure of Regular Languages Under Kleene Star Using NFAs

Theorem. The class of regular languages is closed under the Kleene star operation.

Proof.

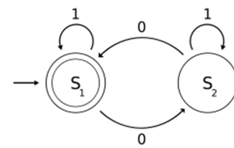


Closure of Regular Languages Under Kleene Star Using NFAs

Theorem. The class of regular languages is closed under the Kleene star operation.

Proof. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1*.



NFA with Single Accept State

Prove that every NFA can be converted to an equivalent one that has a single accept state.