Nondeterminism
Nondeterminism

- The “computation path” is not unique given an input
- Usually easier to design an nondeterministic automata
- This is not a real model, so why should we care?
Relaxing the Rules

Deterministic Finite Automaton (DFA)

Nondeterministic Finite Automaton (NFA)
How Does That Compute?

Deterministic computation:
- start
- ...
- accept or reject

Nondeterministic computation:
- reject
- ...
- accept
For Example

$N_1$

Input: 010110
Another Example
Formally

A **nondeterministic finite automaton** is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where

- \(Q\) is a finite set called the **states**,
- \(\Sigma\) is a finite set called the **alphabet**,
- \(\delta : Q \times \Sigma \rightarrow \mathcal{P}(Q)\) is the **transition function**, where \(\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}\)
- \(q_0 \in Q\) is the **start state**, and
- \(F \subseteq Q\) is the set of **accept states**.
Nondeterminism is Your Friend

Build an NFA that recognizes the language
\[ L = \{ w \mid w \text{ is a string of } a\text{s and } b\text{s that starts and ends with the same symbol and contains at least two symbols} \}. \]
Nondeterminism is Your Friend

Build an NFA that recognizes the language
\[ L = \{ w \mid w \text{ is a string of } 0\text{s and } 1\text{s that starts with } 010 \text{ or ends with } 110 \}. \]

**Hint:** Think of the two “parts” separately and try to glue them using nondeterminism:
- Strings that start with 010
- Strings that end with 110
Nondeterminism is Your Friend

Build an NFA that recognizes the language

\[ L = \{ w \mid w \text{ is a string of 0s and 1s that has a 1 in the 3rd position from the end} \}. \]
Nondeterminism is Your Friend

Build an NFA that recognizes the language
\( L = \{ w \mid w \text{ is a string of 0s and 1s that has a 1 in the 3rd position from the end} \} \).

The DFA for this language would look like this!
Closure Properties of Languages Recognized by NFA
Concatenation

**Theorem.** The class of languages recognized by NFAs is closed under concatenation.

**Proof.**
Union

**Theorem.** The class of languages recognized by an NFA is closed under union.

**Proof.**
Kleene Star

Let $A$ be a language. We define $A^* = \{ x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A \}$.

Let $A = \{0, 1\}$, let $B = \{ w \mid w \text{ is a string of 0s and 1s containing an even number of 1s} \}$, and let $C = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s} \}$.

What are $A^*$, $B^*$, and $C^*$?
Kleene Star

**Theorem.** The class of languages recognized by NFAs is closed under Kleene star.

**Proof.**