Regular Expressions

Sipser: Section 1.3 pages 63 - 69

Regular Expressions

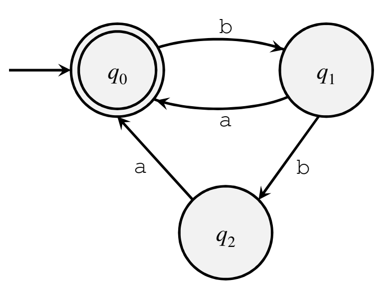
$(0 \cup 1)0^*$

$(0 \cup 1)^*$

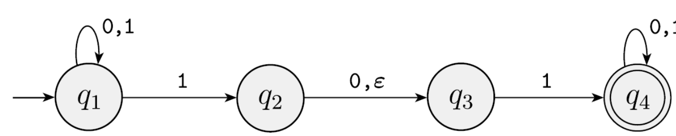
Σ^*1

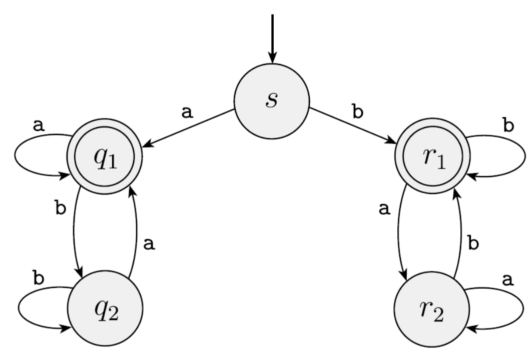
R^+

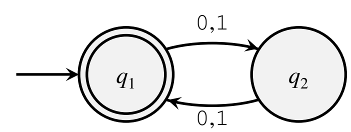
Long Ago in a Place Not Far Away



Old Home Week







Regular Expressions

Definition. Say that R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions,
6. $(R_1)^*$, where R_1 is a regular expression.

Working with Regular Expressions

0^*10^* = { w | }

= { w | w is a string of odd length }

$(0 \cup \varepsilon)(1 \cup \varepsilon)$ =

$(01)^*\emptyset$ =

$(+ \cup - \cup \varepsilon)(DD^* \cup DD^*.D^* \cup D^*.DD^*)$ =
where $D = \{0,1,2,3,4,5,6,7,8,9\}$

Identities

Let R be a regular expression.

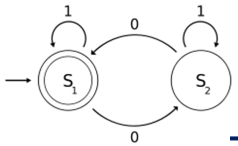
- $R \cup \emptyset =$
- $R \circ \varepsilon =$
- $R \cup \varepsilon =$
- $R \circ \emptyset =$

Regular Expressions and NFAs

Theorem. *A language is regular if and only if some regular expression describes it.*

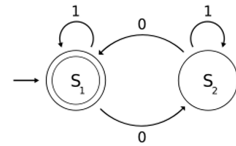
Proof. ($\Leftarrow$)

1. If $a \in \Sigma$, then a is regular.
2. ε is regular.
3. $\emptyset$ is regular.
4. If R_1 and R_2 are regular, then $(R_1 \cup R_2)$ is regular.
5. If R_1 and R_2 are regular, then $(R_1 \circ R_2)$ is regular.
6. If R_1 is regular, then $(R_1)^*$ is regular.



Proof in Action

Build an NFA that recognizes the regular expression: (ab ∪ a)*



Proof in Action

Build an NFA that recognizes the regular expression:

a(a ∪ b)*a