Nonregular Languages

Sipser: Section 1.4 pages 77 - 82

How Do We Know What We Are Missing?

Regular languages may be specified either by regular expressions or by deterministic or nondeterministic finite automata.

How do we show a language is not regular?

Bounded Memory

Since finite automata are not allowed to back up, the amount of memory required to determine whether or not a string is in the language must be bounded.

For example, consider $L = \{0^n 1^n : n \geq 0\}$.

That is a Thought, Not a Proof!

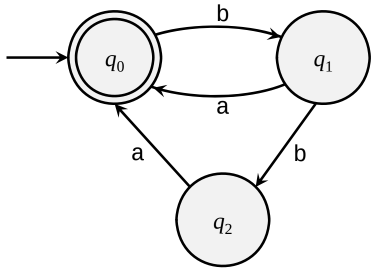
$C = \{w \mid w$ has an equal number of 0s and 1s$\}$.

$D = \{w \mid w$ has an equal number of occurrences of 01 and 10 as substrings$\}$.

Look at it Another Way

From the specification point of view, a regular language is infinite if and only if its corresponding regular expression contains a Kleene star.

Kleene stars correspond to loops in finite automata.

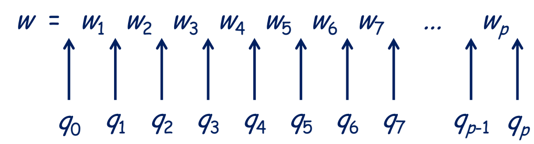


Both Kleene stars and loops give rise to simple repetitive patterns in the language.

The Pigeonhole Principle

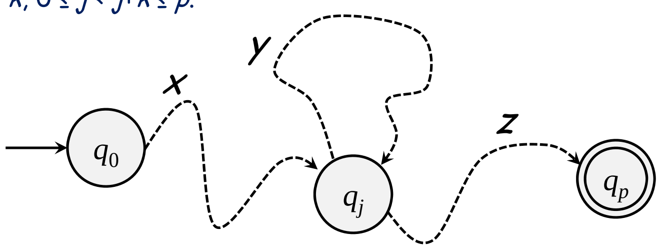
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting an infinite language L. Suppose $|Q| = p$.

Let $w = w_1 w_2 \cdots w_p \in L$.



Machine Loops

Let q_j be the first repeated state, that is, $q_j = q_{j+k}$ for some k, $0 \le j < j+k \le p$.



Where $w = w_1 w_2 \cdots w_p = xyz$,
$x = w_1 w_2 \cdots w_j$ $\quad y = w_{j+1} \cdots w_{j+k}$ $\quad z = w_{j+k+1} \cdots w_p$

We conclude that $xy^i z \in L$ for all $i \ge 0$.

The Pumping Lemma

Theorem. If A is a regular language, then there is a number p (the pumping length) where if w is a string of length at least p, then $w = xyz$, such that

1. for each $i \ge 0$, $xy^i z \in A$,
2. $|y| > 0$, and
3. $|xy| \le p$.

Deciding Regularity

Is $L = \{ 0^n 1^n : n \geq 0 \}$ regular? If so, then there are strings x, y, and z such that $xy^iz \in L$ for all $i \geq 0$. What does y look like?

String y consists entirely of 0s?

String y consists entirely of 1s?

String y consists of both 0s and 1s?

Combining Results

Is $C = \{ w \mid w$ has an equal number of 0s and 1s $\}$ a regular language?

Picking the Right Substring to Pump

Is $PAL = \{ w \in \{0, 1\}^* : w$ is a palindrome $\}$ a regular language?