**Nonregular Languages**

Regular languages may be specified either by regular expressions or by deterministic or nondeterministic finite automata.

How do we show a language is not regular?

**Bounded Memory**

Since finite automata are not allowed to back up, the amount of memory required to determine whether or not a string is in the language must be bounded.

For example, consider $L = \{0^n 1^n : n \geq 0\}$.

That is a Thought, Not a Proof!

$c = \{w \mid w \text{ has an equal number of 0s and 1s}\}$.

$d = \{w \mid w \text{ has an equal number of occurrences of 01 and 10 as substrings}\}$. 

$Sipser$: Section 1.4 pages 77 - 82
**Look at it Another Way**

From the specification point of view, a regular language is infinite if and only if its corresponding regular expression contains a Kleene star.

Kleene stars correspond to loops in finite automata.

Both Kleene stars and loops give rise to simple repetitive patterns in the language.

**The Pigeonhole Principle**

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting an infinite language $L$. Suppose $|Q| = p$.

Let $w = w_1w_2\cdots w_p \in L$.

Let $q_j$ be the first repeated state, that is, $q_j = q_{j+k}$ for some $k$, $0 \leq j < j+k \leq p$.

Where $w = w_1w_2\cdots w_p = xyz$, $x = w_1w_2\cdots w_j$, $y = w_{j+1}\cdots w_{j+k}$, $z = w_{j+k+1}\cdots w_p$

We conclude that $x^iyz \in L$ for all $i \geq 0$.

---

**Machine Loops**

Let $q_j$ be the first repeated state, that is, $q_j = q_{j+k}$ for some $k$, $0 \leq j < j+k \leq p$.

---

**The Pumping Lemma**

Theorem. If $A$ is a regular language, then there is a number $p$ (the pumping length) where if $w$ is a string of length at least $p$, then $w = xyz$, such that

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$. 

We conclude that $x^iyz \in L$ for all $i \geq 0$. 

---
Deciding Regularity

Is $L = \{ 0^n 1^n : n \geq 0 \}$ regular? If so, then there are strings $x, y, \text{ and } z$ such that $x y^i z \in L$ for all $i \geq 0$. What does $y$ look like?

String $y$ consists entirely of 0s?

String $y$ consists entirely of 1s?

String $y$ consists of both 0s and 1s?

Combining Results

Is $C = \{ w \mid w \text{ has an equal number of 0s and 1s} \}$ a regular language?

Picking the Right Substring to Pump

Is $PAL = \{ w \in \{0, 1\}^* : w \text{ is a palindrome} \}$ a regular language?