Problem 1. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. Assume the alphabet is \{0, 1\}.

(a) The language \{w \mid w \text{ contains the substring 0101} \} with five states.

(b) The language \{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s} \} with six states.

(c) The language \{\varepsilon\} with one state.

(d) The language \{w \mid w \text{ contains a pair of 1s separated by an odd number of symbols} \} with four states.

Problem 2. Say whether the following statements are true or false. If it is true, give a brief explanation. If it is false, present a counterexample.

(a) For any NFA \(N\), swapping the accept and nonaccept states will create an NFA \(N'\) such that \(L(N') = \overline{L(N)}\), i.e., the language recognized by \(N'\) is the complement of the language recognized by \(N\).

(b) The class of languages recognized by NFAs is closed under the complement operation.

Problem 3. Use the construction from Lecture 6 to convert the nondeterministic finite automaton below to an equivalent deterministic finite automaton.
Problem 4. For languages $A$ and $B$ with the same alphabet $\Sigma$, let the perfect shuffle of $A$ and $B$ be the language $\text{PS}(A, B)$, defined as

$$\text{PS}(A, B) = \{ w | w = a_1b_1 \ldots a_kb_k \text{ where } a_1a_2\ldots a_k \in A, b_1b_2\ldots b_k \in B, \text{with each } a_i, b_i \in \Sigma \}.$$

Show that the class of regular languages is closed under perfect shuffle.

Problem 5. Recall for any string $w = w_1w_2 \ldots w_n$, the reverse of $w$, written $w^R$, is the string $w$ in reverse order, $w_n \ldots w_2w_1$. For any language $A$, let $A^R = \{ w^R | w \in A \}$. Show that if $A$ is regular, so is $A^R$.

Hint: Use the fact that NFAs and DFAs are equivalent and thus, to show that the language $A^R$ is regular, it is sufficient to construct an NFA recognizing it. Consider the DFA $M$ that recognizes $A$ and construct an NFA $N$ for $A^R$ using $M$ and $\varepsilon$-transitions. Should $N$ have the same number of states as $M$ or a bit more? How do we define the transition function of $N$, given the transition function of $M$?