CS240 Supplemental Practice - Gates

1) For the following circuit, answer the following questions

a. Write out its truth table:

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | Out |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

b. What is the equivalent, unsimplified Boolean expression? In other words, what is the Boolean algebra translation of the circuit above?
$(\mathrm{A}+\mathrm{B})^{\prime}+\left(\mathrm{A}^{\prime} \mathrm{B}^{\prime}\right) \mathrm{C}$
c. What is the sum of products expression of the truth table?

A'B'C + A'B'C'
d. Simplify your answer from part b using Boolean algebra laws. As a sanity check, you should be able to simplify your expression from part c and arrive at the same answer.
(A + B) ${ }^{\prime}+\left(\mathrm{A}^{\prime} \mathrm{B}^{\prime}\right) \mathrm{C}$ - Original expression from part b
A'B' + (A'B')C - DeMorgan's Law
$A^{\prime} B^{\prime}$ - Absorption
A'B'C + A'B'C' - Original expression from part c A'B' - Combining

Extra practice: answer the same questions above with this circuit

a. Write out its truth table:

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | Out |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

b. What is the equivalent, unsimplified Boolean expression? In other words, what is the Boolean algebra translation of the circuit above?
$(A B)^{\prime}+B^{\prime} C$
c. What is the sum of products expression of the truth table?
$\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}+\mathrm{A}^{\prime} \mathrm{BC} C^{\prime}+\mathrm{A}^{\prime} \mathrm{BC}+\mathrm{AB}^{\prime} \mathrm{C}^{\prime}+\mathrm{AB}{ }^{\prime} \mathrm{C}$
d. Simplify your answer from part b using Boolean algebra laws. As a sanity check, you should be able to simplify your expression from part $c$ and arrive at the same answer.
$(A B)^{\prime}+B^{\prime} C-$ Original expression from part $b$
$A^{\prime}+B^{\prime}+B^{\prime} \mathrm{C}-$ DeMorgan's
$A^{\prime}+B^{\prime}$ - Absorption
$\underline{A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B^{\prime} C}+A^{\prime} B C^{\prime}+A^{\prime} B C+A B^{\prime} C^{\prime}+A B^{\prime} C-$ Original expression from part C
$\mathrm{A}^{\prime} \mathrm{B}^{\prime}+\underline{\mathrm{A}^{\prime} \mathrm{BC} C^{\prime}+\mathrm{A}^{\prime} \mathrm{BC}}+\mathrm{AB} \mathrm{A}^{\prime}+\mathrm{AB}{ }^{\prime} \mathrm{C}-$ Combining
$A^{\prime} B^{\prime}+A^{\prime} B+A^{\prime} C^{\prime}+A B^{\prime} C-$ Combining
$\underline{A^{\prime}} \mathrm{B}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{AB}$ ' - Combining
$\underline{A^{\prime}+A B^{\prime}}-$ Combining
$\left(A^{\prime}+A\right)\left(A^{\prime}+B^{\prime}\right)-$ Distributive
(1)(A' $A^{\prime}$ ') - Inverse
$A^{\prime}+B^{\prime}$ - Identity
2) For the Boolean expression: $\left(A^{\prime} A+\left(B C^{\prime}\right)^{\prime}\right)^{\prime}+B$
a. Draw the unsimplified circuit:

b. Write out its truth table:

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | Out |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

c. Simplify or shorten the expression step-by-step using Boolean algebra laws, and write the corresponding laws next to each step
( $\left.\mathrm{A}^{\prime} \mathrm{A}+(\mathrm{BC})^{\prime}\right)^{\prime}+\mathrm{B}-$ Original expression
$\left(0+\left(B C^{\prime}\right)^{\prime}\right){ }^{\prime}+\mathrm{B}$ - Inverse
((BC')')' + B - Identity
(BC') + B - Negation
B - Absorption

Extra practice: answer the questions above with this Boolean algebra expression (A + ABC')B
a. Draw the unsimplified circuit:

b. Write out its truth table:

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | Out |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

c. Simplify or shorten the expression step-by-step using Boolean algebra laws, and write the corresponding laws next to each step
$(A+A B C ') B$ - Original expression
AB - Absorption

