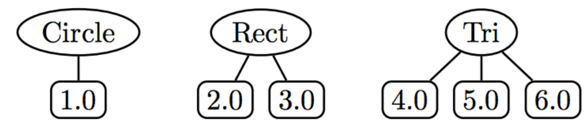
Sum-of-Product Datatypes in SML

CS251 Programming Languages
Fall 2016, Lyn Turbak

Department of Computer Science
Wellesley College

Motivating example: geometric figures

Suppose we want to represent geometric figures like circles, rectangles, and triangles so that we can do things like calculate their perimeters, scale them, etc. (Don't worry about drawing them!)

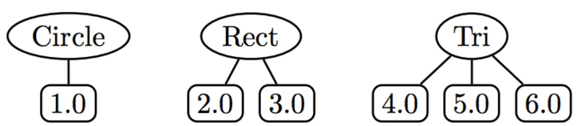


These are so-called **sum of products** data:

- Circle, Rec, and Tri are tags that distinguish which one in a sum
- The numeric children of each tag are the product associated with that tag.

How would you do this in Java? In Python?

SML's `datatype` for Sum-of-Product types




```
datatype figure =
  Circ of real (* radius *)
| Rect of real * real (* width, height *)
| Tri of real * real * real (* side1, side2, side3 *)

val figs = [Circ 1.0, Rect (2.0,3.0), Tri(4.0,5.0,6.0)]
    (* List of sample figures *)

val circs = map Circ [7.0, 8.0, 9.0]
    (* List of three circles *)
```


Functions on datatype via pattern matching

```
(* Return perimeter of figure *)
fun perim (Circ r) = 2.0 * Math.pi * r
  | perim (Rect(w,h)) = 2.0 * (w + h)
  | perim (Tri(s1,s2,s3)) = s1 + s2 + s3

(* Scale figure by factor n *)
fun scale n (Circ r) = Circ (n * r)
  | scale n (Rect(w,h)) = Rect (n*w, n*h)
  | scale n (Tri(s1,s2,s3)) = Tri (n*s1, n*s2, n*s3)
```

```
- val perims = map perim figs
val perims = [6.28318530718,10.0,15.0] : real list

- val scaledFigs = map (scale 3.0) figs
val scaledFigs = [Circ 3.0,Rect (6.0,9.0),
                  Tri (12.0,15.0,18.0)] : figure list
```

Options

SML has a built-in `option` datatype defined as follows:

```
datatype 'a option = NONE | SOME of 'a
```

```
- NONE
val it = NONE : 'a option

-  SOME 3;
val it = SOME 3 : int option

- SOME true;
val it = SOME true : bool option
```


Sample Use of Options

```
- fun into_100 n = if (n = 0) then NONE else SOME (100 div n);
val into_100 = fn : int -> int option

- List.map into_100 [5, 3, 0, 10];
val it = [SOME 20,SOME 33,NONE,SOME 10] : int option list

- fun addOptions (SOME x) (SOME y) = SOME (x + y)
=    | addOptions (SOME x) NONE = NONE
=    | addOptions NONE (SOME y) = NONE
=    | addOptions NONE NONE = NONE;
val addOptions = fn : int option -> int option -> int option

- addOptions (into_100 5) (into_100 10);
val it = SOME 30 : int option

- addOptions (into_100 5) (into_100 0);
val it = NONE: int option
```


Options and `List.find`

```
(* List.find : ('a -> bool) -> 'a list -> 'a option *)
- List.find (fn y => (y mod 2) = 0) [5,8,4,1];
val it = SOME 8 : int option

- List.find (fn z => z < 0) [5,8,4,1];
val it = NONE : int option
```


Thinking about options

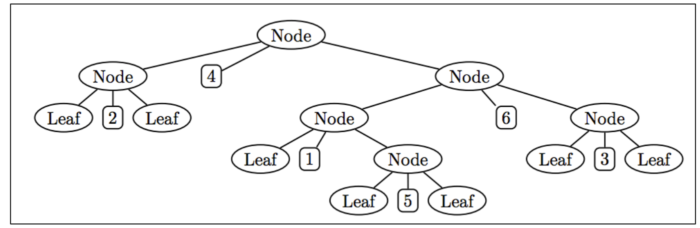
What problem does option solve?

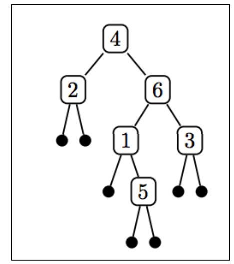
How is the problem solved in other languages?

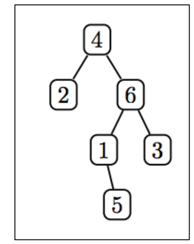
Creating our own list datatype

```
datatype 'a mylist = Nil | Cons of 'a * 'a mylist

val ints = Cons(1, Cons(2, Cons(3, Nil))) (* : int mylist *)

val strings = Cons("foo", Cons ("bar", Cons ("baz", Nil)))
(* : strings mylist *)

fun myMap f Nil = Nil
  | myMap f (Cons(x,xs)) = Cons(f x, myMap f xs)
(* : ('a -> 'b) -> 'a mylist -> 'b mylist *)

val incNums =  myMap (fn x => x + 1) ints
(* val incNums= Cons (2,Cons (3,Cons (4,Nil))) : int mylistval *)

val myStrings = myMap (fn s => "my " ^ s) strings
(* val myStrings = Cons ("my foo", Cons ("my bar", Cons ("my
baz",Nil))): string mylist *)
```

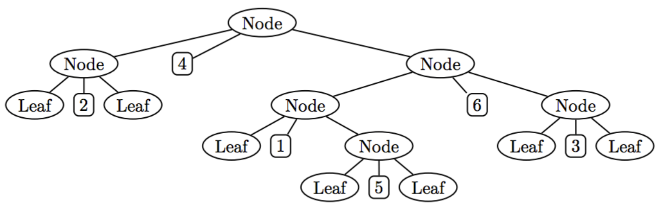

Binary Trees







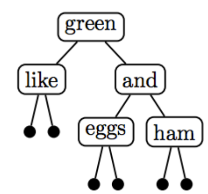
SML `bintree` datatype for Binary Trees




```
datatype 'a bintree =
    Leaf
  | Node of 'a bintree * 'a * 'a bintree
    (* left subtree, value, right subtree *)
```

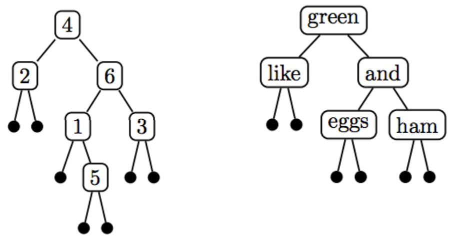
```
val int_tree= Node(Node(Leaf,2,Leaf),
                   4,
                   Node(Node(Leaf, 1, Node(Leaf, 5, Leaf)),
                        6,
                        Node(Leaf, 3, Leaf)))
```


`bintree` can have any type of element




```
val string_tree = Node(Node (Leaf,"like",Leaf),
                       "green",
                       Node (Node (Leaf,"and",Leaf),
                             "eggs",
                             Node (Leaf,"ham",Leaf)))
```

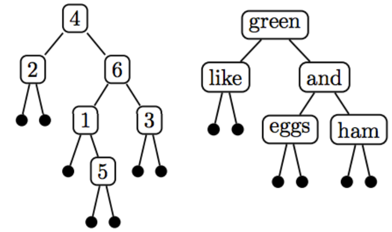
Counting nodes in a binary tree




```
fun num_nodes Leaf = 0
  | num_nodes (Node(l,v,r)) = 1 + (num_nodes l) + (num_nodes r)
```

```
- num_nodes int_tree;
val it = 6 : int

- num_nodes string_tree;
val it = 5 : int
```

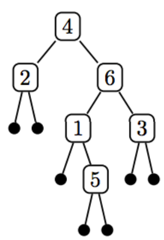

Your turn: height




```
(* val height = fn : 'a bintree -> int *)
(* Returns the height of a binary tree. *)
(* Note: Int.max returns the max of two ints *)

fun height Leaf = 0
  | height (Node(l,v,r)) = 1 + Int.max(height l, height r)
```

```
- height int_tree;
val it = 4 : int

- height string_tree;
val it = 3 : int
```


Your turn: sum_nodes

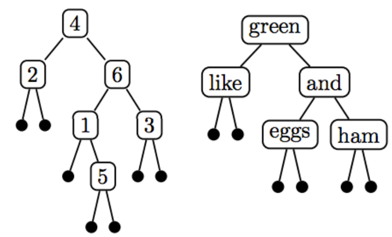



```
(* val sum_nodes = fn : int bintree -> int *)
(* Returns the sum of node values in binary tree of ints *)

fun sum_nodes Leaf = 0
  | sum_nodes (Node(l,v,r)) = (sum_nodes l) + v + (sum_nodes r)
```

```
- sum_nodes int_tree;
val it = 21 : int
```


Your turn: inlist

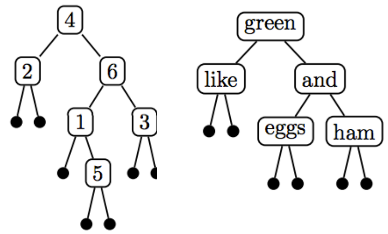
This returns a list of elements as they are Encountered in an **in-order** traversal of a tree. We could also list them via a **pre-order** or **post-order** traversal.




```
(* val inlist = fn : 'a bintree -> 'a list *)
(* Returns a list of the node values in in-order *)

fun inlist Leaf = []
  | inlist (Node(l,v,r)) = (inlist l) @ [v] @ (inlist r)
```

```
- inlist int_tree;
val it = [2,4,1,5,6,3] : int list

- inlist string_tree;
val it = ["like","green","eggs","and","ham"] : string list
```

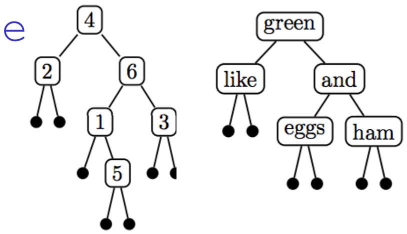
Your turn: map_tree




```
(* val map_tree = fn : ('a -> 'b) -> 'a bintree -> 'b bintree *)
(* maps function over every node in a binary tree *)

fun map_tree f Leaf = Leaf
  | map_tree f (Node(l,v,r)) = Node(map_tree f l, f v, map_tree f r)
```

```
- map_tree (fn x => x*2) int_tree;
val it =  Node (Node (Leaf,4,Leaf),8,
                Node (Node (Leaf,2,Node (Leaf,10,Leaf)),12,
                      Node (Leaf,6,Leaf))) : int bintree

-  map_tree (fn s => String.sub(s,0)) string_tree;
val it =  Node (Node (Leaf,#"l",Leaf),#"g",
                Node (Node (Leaf,#"e",Leaf),#"a",
                      Node (Leaf,#"h",Leaf))) : char bintree
```

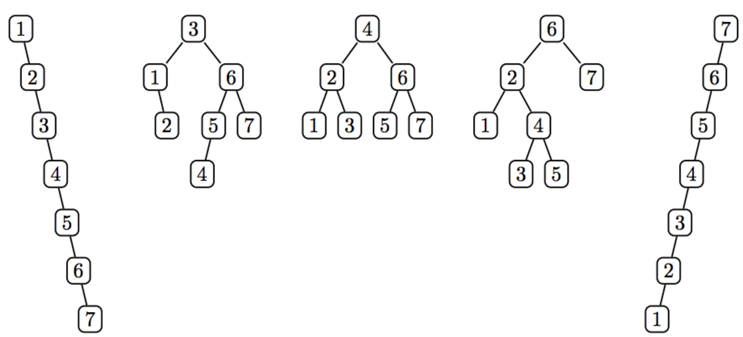

Your turn: fold_tree




```
(* val fold_tree = fn : ('b * 'a * 'b -> 'b) -> 'b
                        -> 'a bintree ->  'b *)
(* binary tree accumulation *)

fun fold_tree comb leafval Leaf = leafval
  | fold_tree comb leafval (Node(l,v,r)) =
    comb(fold_tree comb leafval l, v, fold_tree comb leafval r)
```

```
- fold_tree (fn (lsum,v,rsum) => lsum + v + rsum) 0 int_tree;
val it = 21 : int

- fold_tree (fn (lstr,v,rstr) => lstr ^ v ^ rstr) " " string_tree;
val it = " like green eggs and ham " : string
```


Binary Search Trees (BSTs) on integers



You turn: Binary Search Tree insertion

```
fun singleton v = Node(Leaf, v, Leaf)

(* val insert: 'a bintree -> 'a -> 'a bintree *)
fun insert x Leaf = singleton x
  | insert x (t as (Node(l,v,r))) =
      if x = v then t
      else if x < v then Node(insert x l, v, r)
      else Node(l, v, insert x r)

fun listToTree xs = foldl (fn (x,t) => insert x t)
                          Leaf xs
```

```
- val test_bst = listToTree [4,2,3,6,1,7,5];
val test_bst = Node (Node (Node (Leaf,1,Leaf),
                           2,
                           Node (Leaf,3,Leaf)),
                     4,
                     Node (Node (Leaf,5,Leaf),
                           6,
                           Node (Leaf,7,Leaf))) : int bintree
```

Your turn: Binary Search Tree membership

```
(val member: 'a -> 'a bintree -> bool *)
fun member x Leaf = false
  | member x (Node(l,v,r)) =
      (x = v) orelse member x l orelse member x r
```

```
- val test_member = map (fn i => (i, member i
test_bst)) [0,1,2,3,4,5,6,7,8];
val it = [(0,false),(1,true),(2,true),(3,true),
(4,true),(5,true),(6,true),(7,true), (8,false)] :
(int * bool) list
```


Balanced Trees (PS7)

BSTs are not guaranteed to be balanced.

But there are other tree data structures that do guarantee balance:
AVL trees, Red/Black trees, 2-3 trees, 2-3-4 trees.

In PS7 you will experiment with 2-3 trees.