CS251 Programming Languages Prof. Lyn Turbak Wellesley College

FINAL EXAM REVIEW PROBLEMS

The CS251 final is a self-scheduled final exam held during the normal final exam period. The exam is open book; you may refer to the textbook, class handouts, your notes, and whatever additional materials you find useful. However, you may **not** use a computer during the exam. By the Honor Code, you are **not** allowed to talk to anyone about the details of the exam before or after taking it, until the final examination period is over.

Here is a list of topics covered in CS251 that are fair game for the final exam:

- programming paradigms: functional, imperative, object-oriented
- **syntax**: abstract syntax trees, free variables, substitution, desugaring
- evaluation models and interpreters: substitution model, environment model
- data: first-class functions, aggregate data programming, sum-of-product datatypes, lazy data
- **scoping**: lexical, dynamic, global; block structure; environment diagrams & closures
- parameter passing: call-by-value, call-by-name, call-by-need, call-by-reference.
- **types**: dynamic vs. static; explicit vs. reconstructed ; monomorphic vs. polymorphic
- **imperative programming**: mutable data, mutable variables, memoization; benefits and drawbacks.
- **control constructs**: tail recursion, non-local exits (label/jump, call-with-current-continuation); exceptions (raise, trap, handle); continuation-passing style.
- **real languages**: Scheme, ML, a smattering of Haskell, C, and Pascal. Note: You will be expected to read and write programs in Scheme and ML. You will be expected to read very simple Haskell, C, and Pascal programs, but will not be expected to write them.
- toy languages:
 - INTEX = integer literals + arithmetic operations + program parameters
 - BINDEX = INTEX + local binding (bind)
 - IBEX = BINDEX + boolean literals + relational & arithmetic ops + if
 - FOFL = BINDEX + top-level recursive first-order function declarations + list primitives
 - FOBS = FOFL + local recursive first-order function declarations (funrec)
 - HOFL = FOBS + abstractions (allowing higher-order functions) + bindrec (merges function and variable namespaces)
 - HOFLEMT = HOFL with explicit monomorphic types.
 - HOFLEPT = HOFL with explicit polymorphic types.
 - HOFLIMT = HOFL with implicit monomorphic types.
 - HOFLIPT = HOFL with implicit polymorphic types.
 - HOILEC = HOFL + explicit mutable cells + sequencing + input/output
 - HOILIC = HOILIC with implicit mutable cells for every variable; these are assigned to via <-.

Note: The following problems are intended to help you review material for the final exam. They are not necessarily indicative of the kinds of questions that will be asked on the exam (i.e., some review questions are more difficult/time consuming than what would be on an exam.) They also do not cover all of the above topics.

Problem 1: ML Types

Consider the following sequence of function declarations in the ML language:

```
fun test1 (x, f, g) = (x, f(x), g(x))
fun test2 (x, f, g) = (x, f(x), g(f(x)))
fun test3 (x, f, g) = (x, f(x), g(f(x)), f(g(x)))
fun test4 (x, f, g) = (x, f(x), g(x, f(x)))
fun test5 (x, f, g) = (x, f(x), g(f(x), f(g(x))))
fun test6 (x, f, g) = (x, f(x), g(x, f(g(x))))
```

Part a. For each of the above function declarations, write down the type that ML would reconstruct for the function. If ML would not be able to reconstruct a type for a function, say so and explain why.

Part b.

- (1) Define a curried version of the test1 function named test1-curried.
- (2) Give the type of test1-curried.
- (3) Below is an expression using test1. Show how to rewrite it using test1-curried:

test1(3, fn y => y * 2, fn z => z > 0)

Part c. Below is a curry2 function curries any function whose argument is a tuple of two values. What is the type of curry2?

```
fun curry2 f = (fn x => (fn y => f(x,y)))
```

Part d. Define an uncurry2 function that is the inverse of curry2. That is, for any curried function f of two arguments, curry2(uncurry2(f)) should be indistinguishable from f; and for any uncurried function g of two arguments, uncurry2(curry2(g)) should be indistinguishable from g.

Part e. While Scheme and ML are similar in many respects, Scheme is a dynamically typed language while ML is a statically typed language. Briefly discuss the advantages and disadvantages of static typing vs. dynamic typing.

Part f. While both ML and Java are statically typed languages, there are some key differences between the languages. Briefly describe the main differences.

Problem 2: Environment Diagrams and Mutation

Consider the following procedures in an imperative call-by-value statically-scoped Scheme:

```
(define make-updater
  (let ((n 0))
    (lambda (init update)
      (begin
        (set! n (+ n 1))
        (let ((uid n)
              (state init))
          (lambda ()
            (begin (set! state (update uid state))
                   state)))))))
(define try
  (lambda (u n)
    (if (= n 0))
        `()
        (let ((v (u)))
          (cons v (try u (- n 1)))))))
(define test
  (lambda ()
    (let* ((a (make-updater `() cons))
           (b (make-updater 1 *))
           (c (make-updater 0 +)))
      (list (try a 2) (try b 3) (try c 4)))))
```

What is the value of (test)? Draw environment diagrams to justify your answer. You need not draw environment frames for invocations of the try procedure.

Problem 3: Church Pairs

Although HOFL supports lists, it does not support Scheme-like pairs that can glue together any two values. However, it is possible to implement Scheme-like pairs as HOFL functions, as illustrated by the following HOFL program:

When called on two arguments, a and b, cons returns a procedure (call it p for pair) as a result. The pair p is a procedure of one argument, f, that calls f on a and b. The car procedure takes such a pair p and applies it to a function that returns the first of its two arguments, while cdr applies p to a function that returns the second of its two arguments. This representation pairs is called a *Church pair* after its inventor, the logician Alonzo Church.

Part a. Use the substitution model to prove that (car (cons 3 4)) yields 3 for the above definitions of cons and car. (A similar argument would show that (cdr (cons 3 4)) yields 4.)

Part b. Use the environment model to prove that (car (cons 3 4)) yields 3 for the above definitions of cons and car.

Part c. Would the above definitions work in a dynamically scoped version of HOFL? Explain.

Part d. Translate the above HOFL program into the explicitly-typed HOFLEPT language. You will need to make each of cons, car, and cdr polymorphic. The type of cons should be:

Part e. In Scheme, cons, car, and cdr are not only used to define general pairs, but can also be used to define lists. Is the same true in (untyped) HOFL? How about in explicitly typed HOFLEPT?

Part f. In HOILEC, the imperative version of HOFL with explicit cells, the above definitions can be extended to support Scheme's pair mutation operators set-car! and set-cdr!. Show how this can be done by filling out the the expressions <fill_i> below.

Problem 4: Explicit Typing

For each of the following two programs in the implicitly-typed HOFLIPT language, translate the program into the explicitly-typed HOFLEPT language.

```
(program (n)
  (bindrec ((even? (abs (n)
                     (if (= n 0))
                          #t
                          (odd? (- n 1)))))
            (odd? (abs (n)
                    (if (= n 0))
                        #f
                        (even? (- n 1)))))
   (prepend (even? 5)
     (prepend (odd? 5)
       (empty)))))
(program (hi)
  (bindrec ((map (abs (f lst))
                   (if (empty? lst)
                       (empty)
                       (prepend (f (head lst))
                                 (map f (tail lst))))))
            (from-to (abs (lo)
                       (if (> lo hi)
                           (empty)
                           (prepend lo (from-to (+ lo 1))))))
    (bind test-list (from-to 1)
      (prepend (map (abs (n) (prepend n (empty)))
                     (map (abs (x) (* x x)) test-list))
               (prepend (map (abs (b)
                                (if b
                                    (prepend 1 (empty))
                                    (prepend 0 (empty))))
                              (map (abs (y) (= (mod y 2) 0)))
                                   test-list))
                         (prepend (map (abs (z)
                                         (prepend z
                                            (prepend (* 2 z)
                                             (empty))))
                                        test-list)
                                  (empty))))))))
```

Problem 5: Non-local Exits

A *binary tree* is either (1) a *leaf* or (2) the result of applying the node constructor to a left binary tree and a right binary tree. The leaf? predicate determines if a value is a leaf (non-node), and the selectors left and right extract the left and right subtrees of a binary tree.

Assume that append is a procedure that takes two lists and returns a new list containing all of the elements of the first followed by all of the elements of the second. E.g.:

(append '(a b c) '(d e)) returns the list (a b c d e)

Assume that postpend is a procedure that takes a list L and a value V and returns a new list containing all of the elements of L followed by V. E.g.:

(postpend '(a b c) d) returns the list (a b c d)

Consider the following fringe procedure, which is written in a version of call-by-value, staticallyscoped Scheme supporting the label and jump constructs:

Part a. For each of the three expressions in the following table, indicate the value of the expression. Assume that the operand expressions of a function application are evaluated in left-to-right order.

Expression	Value
(fringe (node (node `a `b) (node `c `d)))	
(fringe (node (node `a 2) (node `c `d)))	
(fringe (node (node 'a 'b) (node 3 'd)))	
(fringe (node (node 'a 2) (node 3 'd)))	

Part b. Give an English specification for fringe.

Part c. Write an function fringe-cps that behaves the same as fringe, but uses explicit continuations rather than the implicit ones used by fringe.

Part d. Describe the difficulties that would be encountered in implementing fringe without using explicit or implicit continuations.

Problem 6: Parameter Passing

Consider the following expression:

For each of the following parameter-pasing mechanisms, indicate the value of the above expression in a version of lexically-scoped Scheme using that parameter-passing mechanism. Assume that all operands are evaluated in left-to-right order.

Parameter-Passing Mechanism	Value of sample expression
Call-by-value	
Call-by-reference	
Call-by-name	
Call-by-need	

Problem 7: Parameter Passing

Consider the following expression:

For each of the following parameter-pasing mechanisms, indicate the value of the above expression in a version of Scheme using that parameter-passing mechanism. Assume that all operands are evaluated in left-to-right order.

Parameter-Passing Mechanism	Value of sample expression
Call-by-value	
Call-by-reference	
Call-by-name	
Call-by-need	

Problem 8: Desugaring

One way to define an or construct is as a user-defined procedure:

(define orl (lambda (a b) (if a a b))

An alternative way to define an or construct is via syntactic sugar:

 $(or2 E_1 E_2)$ desugars to $(let ((I E_1)) (if I I E_2))$; assume I fresh

Part a. For each of the following parameter passing mechanisms in an *imperative* version of statically-scoped Scheme, explain your answer to the following question:

Are (or1 $E_1 E_2$) and (or2 $E_1 E_2$) interchangeable for all expressions E_1 and E_2 ?

• call-by-value

- call-by-name
- call-by-need

Part b. The desugaring for or2 has the side condition "assume I fresh". What could go wrong with this desugaring if the side condition were omitted?

Part c. What are the advantages of defining a language construct via desugaring rather than adding it as a kernel construct of the language?

Problem 9: Block Structure

Translate each of the following two block-structured top-level FOBS function declarations into an equivalent collection of FOFL (non-block-structured) function declarations. Recall that a collection of top-level fun declarations in FOBS desugar into a funce, so that each of the following function declarations is recursively defined.

```
(fun index-of-bs (elt lst)
      (funrec ((index-loop (i L)
                 (if (empty? L)
                     -1
                     (if (= elt (head L))
                          (index-loop (+ i 1) (tail L))))))
        (index-loop 1 lst)))
(fun cartesian-product-bs (lst1 lst2)
      (funrec ((prod (lst)
                 (if (empty? lst)
                     (empty)
                     (let ((elt (head lst)))
                        (funrec ((duple (b)
                                   (prepend elt
                                     (prepend b
                                       (empty)))))
                                 (map-duple (L)
                                   (if (empty? L)
                                       (empty)
                                       (prepend (duple (head L))
                                                 (map-duple (tail L))))))
                          (append (map-duple lst2)
                                  (prod (tail lst)))))))
         (prod lst1)))
```

Problem 10: Static vs. Dynamic Scope

Part a. Consider the following definitions in call-by-value Scheme:

```
(define (raise-to n)
  (lambda (x) (expt x n))) ; (expt x n) computes x<sup>n</sup>
(define (sum proc n limit)
  (if (> n limit)
      0
      (+ (proc n)
           (sum proc (+ n 1) limit))))
```

For each of the following two scoping mechanisms, indicate the value of the expression (sum (raise-to 2) 1 3) in a version of Scheme using that scoping mechanism:

Scoping Mechanism	Value of (sum (raise 2) 1 3)
Lexical	
Dynamic	

Part b. Can a language be lexically scoped without being block structured? Briefly explain your answer.

Problem 11: Scoping

in the enclosing lexical context.

Renowned naming expert Dan Emmet Schoop is experimenting with a new binding construct called fluid-bind. Dan writes the following specification for his construct:

```
(fluid-bind I E_def E_body)
Temporarily assign I to the value of E_def during the evaluation of E_body and then reset I
to its original value. Returns the value of E_body. Signals an error if I is not already bound
```

Dan adds his construct to an interpreter for statically-scoped imperative HOILIC by extending the env-eval function with the following clause:

Part a. Determine the values of the following two expressions that use fluid-bind:

Expression	Value
(bind a 1	
(bind f (abs (x) (+ x a))	
(+ (fluid-bind a 20	
(f 300))	
(f 4000))))	
(bind a 1	
(+ (fluid-bind a 20	
(seq	
(set! a (+ a 300))	
a))	
a))	

Part b. Dan could have defined fluid-bind as syntactic sugar for other HOILIC constructs. Below, give a desugaring rule that indicates how fluid-bind can be written in terms of other HOILIC constructs. Write your desugaring rule as a rewrite rule whose form is similar to the following desugaring rule for let:

(bindpar ((I Edef) ...) Ebody)
desugars to ((abs (I ...) Ebody) Edef ...)

You should strive to avoid accidental name capture.

Part c. Dan's friend Bud Lojack writes the following number printing program in Dan's extended version of HOILIC.

Bud evaluates (print-nums 0), and watches as the numbers 0, 1, 2, etc. are displayed on the screen. However, Bud is surprised when his procedure eventually crashes due to a lack of stack space. "I thought Scheme was properly tail recursive!" he exclaims. "Why doesn't my procedure behave like an infinite loop?" Kindly explain to Bud why his program has run out of stack space.

Part d Dan points out that in many situations fluid-bind acts like a dynamically scoped version of bind. However, he notes that fluid-bind and a dynamically scoped bind can give different behavior in the case where a non-local exit (such as jump or raise) is encountered during the evaluation of the body expression. Briefly explain what Dan means by this comment.

PROBLEM 12: Scoping

H&R Block Structure, a tax software vendor, has developed a program for computing the cost of taxable items in a *dynamically scoped* imperative call-by-value version of Scheme. Their program includes the following top-level definitions:

```
(define *rate* 0.05)
(define taxed
  (lambda (amount)
    (* amount (+ 1 *rate*))))
(define with-rate
  (lambda (rate thunk)
    (let ((*rate* rate))
        (thunk))))
```

The global variable *rate* represents the default sales tax rate (5%). The procedure taxed uses the global value of *rate* unless it has been shadowed by a local binding of *rate*, such as that made by with-rate. This approach is more convenient than having to pass tax rates as explicit parameters throughout a large program. For example, consider the expression E_{tax} :

```
(+ (taxed 200)
(+ (with-rate 0.075 (lambda () (taxed 1000)))
(taxed 400)))
```

This expression evaluates to 210 + 1075 + 420 = 1705.

a. What is the value of E_{tax} in a statically-scoped version of Scheme? Explain.

b.. H&R Block Structure asks you to port their code to a *lexically-scoped* imperative call-by-value Scheme. Show how to define with-rate in lexically-scoped Scheme so that it has the same behavior as the above with-rate in a dynamically scoped mini-Scheme. *Hint*: use side effects. Also, compare with Problem 11.

Problem 13: Variables and Scoping

Consider the following expression in statically-scoped HOILEC (the Higher-Order Imperative Language with Explicit Cells):

Part a. Circle all of the free variable references in the above expression.

- **Part b.** For each bound variable reference, draw an arrow from the reference to the point where the variable is declared.
- Part c. Suppose that the above expression is evaluated in an environment in which

1. map is the usual higher-order mapping function.

2. all other free variables are initially bound to the number 1.

Give the value of the above expression under each of the following parameter passing mechanisms. If the expression loops, raises an error, or is otherwise undefined, say so.

- call-by-value:
- call-by-name
- call-by-need

Problem 14: The Aggregate Data Style of Programming

Here's a Scheme procedure that prompts the user for a sequence of non-negative integers and returns the percentage of even integers in that sequence. The user indicates the end of the sequence by typing a negative integer:

Assume that the nullary read-int procedure prompts the user (via the prompt int>) for a single integer and returns this integer. Then here's a sample use of even-pct:

```
(even-pct)
int> 3
int> 8
int> 2
int> -1
0.666666 ; Two out of the three integers were even.
```

Part a. Rewrite even-pct as an aggregate data style program in terms of the higher-order procedures generate, map, filter, and foldr. (See Appendix A for definitions of these higher order procedures.) You may *not* assume the existence of a length function for lists; if you need one, you must define it in terms of generate, map, filter, and foldr.

Part b. Briefly describe two advantages of writing even-pct in the aggregate data style vs. the original style.

Part c. Briefly describe two disadvantages of writing even-pct in the aggregate data style vs. the original style.

Part d. Proponents of lazy functional programming languages claim that laziness is essential for programming in the signal processing style. Briefly explain their claim.

PROBLEM 15 : Lazy Data

Let the term *ordered duple* ("orduple" for short) refer to a list of two non-negative integers in which the first integer is less than or equal to the second integer. E.g. $(0 \ 2)$, $(1 \ 2)$ and $(2 \ 2)$ are all orduples, but $(-1 \ 2)$ and $(2 \ 1)$ are not orduples. Orduple a is said to be less than duple b if either

1. (+ (first a) (second a)) is less than (+ (first b) (second b))
or 2. (+ (first a) (second a)) is equal to (+ (first b) (second b))
 but (first a) is less than (first b).

For example, the first nine orduples in order are:

 $(0 \ 0) \ (0 \ 1) \ (0 \ 2) \ (1 \ 1) \ (0 \ 3) \ (1 \ 2) \ (0 \ 4) \ (1 \ 3) \ (2 \ 2)$

Part a. Using Scheme streams, define an infinite sorted stream of all orduples named allorduples. You may use whatever auxiliary procedures you find helpful as part of your definition, including the higher order stream operators in appendix B.

Part b. Pythagorean triples are length-3 lists of the form (a b c) where 0 < a b and $a^2 + b^2 = c^2$. Using all-orduples from Part a and the stream operators from Appendix B, define an infinite stream pythagoreans that contains all Pythagorean triples.

You may assume that the Scheme sqrt function returns an integer when called on a perfect square. That is, (sqrt 25) returns the integer 5, not the floating point number 5.0. The Scheme predicate integer? tests whether a given value is an integer.

Part c. The definition of all-orduples from part a will not work if lists are used in place of streams. Explain why.

Problem 16: Types

a Write a version of the generate procedure from Appendix A in the explicitly typed polymorphic HOFLEPT language presented in class. Recall that cons and '() have the following types:

```
cons : (forall (T) (-> (T (listof T)) (listof T)))
'() : (forall (T) (listof T))
```

b What is the type of the zip procedure from Appendix A expressed in HOFLEPT?

c The type for zip in part b is more restrictive than the type would be for ML's zip function. Briefly explain why this is the case.

Problem 17: Control

Consider the following map2 procedure in a version of Scheme supporting exception handling (via raise/trap/handle):

map2 maps a two-argument procedure over the corresponding elements of two lists:

> (map2 * '(1 2 3) '(4 5 6)) (4 10 18)

If the two lists do not have the same length, map2 raises the length error.

a. One way to handle lists of different lengths is to ignore the elements of the longer list that do not correspond to elements in the shorter one. Below, write a map2-truncate procedure that has this behavior. For example:

```
> (map2-truncate * '(1 2 3) '(4 5 6))
(4 10 18)
> (map2-truncate * '(1 2 3 4 5) '(4 5 6))
(4 10 18)
> (map2-truncate * '(1 2 3) '(4 5 6 7 8))
(4 10 18)
```

map2-truncate should be expressed as an exception handler wrapped around a call to map2.

b. Another way to handle lists of different lengths is to return the symbol failed. Below, write a map2-fail procedure that has this behavior. For example:

```
> (map2-fail * '(1 2 3) '(4 5 6))
(4 10 18)
> (map2-fail * '(1 2 3 4 5) '(4 5 6))
failed
> (map2-fail * '(1 2 3) '(4 5 6 7 8))
failed
```

map2-fail should be expressed as an exception handler wrapped around a call to map2.

c. In a language that supports label and jump constructs in addition to raise and trap, it is possible to simulate handle by using trap in conjunction with label and jump. Show this by writing a procedure map2-fail2 that :

1. behaves exactly like map-fail;

2. is implemented by wrapping a trap (not handle) handler around a call to map 2.

Appendix A: Definitions of Higher-Order List Operations

```
(define zip
  (lambda (lst1 lst2)
    (if (or (null? lst1) (null? lst2))
        '()
        (cons (list (car lst1) (car lst2))
              (zip (cdr lst1) (cdr lst2))))))
(define generate
  (lambda (seed next done?)
    (if (done? seed)
      '()
      (cons seed (generate (next seed) next done?)))))
(define map
  (lambda (f lst)
    (if (null? lst)
      '()
      (cons (f (car lst))
            (map f (cdr lst))))))
(define map2
  (lambda (f lst1 lst2)
    (map (lambda (duple)
           (f (first duple) (second duple)))
         (zip lst1 lst2))))
(define filter
  (lambda (pred lst)
    (if (null? lst)
      '()
      (if (pred (car lst))
        (cons (car lst) (filter pred (cdr lst)))
        (filter pred (cdr lst))))))
(define foldr
  (lambda (binop init lst)
    (if (null? lst)
      init
      (binop (car lst) (foldr binop init (cdr lst))))))
(define foldr2
  (lambda (ternop init lst1 lst2)
    (foldr (lambda (duple result)
             (ternop (first duple) (second duple) result))
           init
           (zip lst1 lst2))))
(define fold)
  (lambda (binop init lst)
    (if (null? lst)
      init
      (foldl binop (binop (car lst) init) (cdr lst)))))
(define foldl2
  (lambda (ternop init lst1 lst2)
    (foldl (lambda (duple result)
             (ternop (first duple) (second duple) result))
           init
           (zip lst1 lst2))))
```

```
(define forall?
  (lambda (pred lst)
    (if (null? lst)
        #t
        (and (pred (car lst))
             (forall? pred (cdr lst))))))
(define forall2?
  (lambda (pred lst1 lst2)
    (forall? (on-duple pred)
             (zip lst1 lst2))))
(define exists?
  (lambda (pred lst)
    (if (null? lst)
        #f
        (or (pred (car lst))
            (exists? pred (cdr lst))))))
(define exists2?
  (lambda (pred lst1 lst2)
    (exists? (on-duple pred)
             (zip lst1 lst2))))
(define some
  (lambda (pred lst)
    (if (null? lst)
        #f
        (if (pred (car lst))
            (car lst)
            (some pred (cdr lst))))))
(define some2
  (lambda (pred lst1 lst2)
    (some (on-duple pred)
          (zip lst1 lst2))))
(define on-duple
  (lambda (f)
    (lambda (duple)
      (f (first duple) (second duple)))))
```

Appendix B: Definitions of Higher-Order Stream Operations

```
(define map-stream2
  (lambda (f strl str2)
    (if (or (stream-null? str1) (stream-null? str2))
        the-empty-stream
        (cons (f (head str1) (head str2))
              (map-stream2 f (tail str1) (tail str2))))))
(define append-streams
  (lambda (str1 str2)
    (if (stream-null? str1)
        str2
        (cons-stream (head str1)
                     (append-streams (tail str1) str2)))))
(define append-streams-delayed
  (lambda (str1 delayed-str2)
    (if (stream-null? str1)
        (force delayed-str2)
        (cons-stream (head str1)
                     (append-streams-delayed (tail str1) delayed-str2)))))
(define append-stream-of-streams
  (lambda (str)
    (if (stream-null? str)
        the-empty-stream
        (append-streams-delayed
          (head str)
          (delay (append-stream-of-streams (tail str)))))))
(define append-map-stream
  (lambda (f str)
    (append-stream-of-streams
     (map-stream f str)
(define filter-stream
  (lambda (pred str)
    (if (stream-null? str)
        the-empty-stream
        (if (pred (head str))
            (cons-stream (head str)
                         (filter-stream pred (tail str)))
            (filter-stream pred (tail str))))))
(define foldr-stream
  (lambda (op init str)
    (if (null? str)
        init
        (op (head str)
            (foldr-stream op init (tail str))))))
(define foldl-stream
  (lambda (op init str)
    (if (stream-null? str)
        init
        (foldl-stream op (op init (head str)) (tail str)))))
```