Translating Closures into FOFL

- Can translate FOBS to FOFL by lifting nested functions to top-level and adding extra arguments.
- Can we translate HOFL closures to FOFL? Yes – via a process called closure conversion.
- Closure conversion can be performed automatically, but it’s often helpful to perform it manually to simulate higher order functions in languages like C and Java.
**sigma Example in HOFL**

(hofl (n) (test n)
  (def (sigma f lo hi)
    (if (> lo hi) 0
      (+ (f lo) (sigma f (+ lo 1) hi))))
  (def (test w)
    (list (sigma sq 1 w)
      (sigma (scale 2) 1 w)
      (sigma (linear 3 4) 1 w)))
  (def (sq x) (* x x))
  (def (scale c) (fun (y) (* c y)))
  (def (linear a b) (fun (z) (+ (* a z) b))))

**Closure-converting sigma in FOFL**

(fofl (n) (test n)
  (def (sigma f lo hi)
    (if (> lo hi) 0
      (+ (applyClosure f lo) (sigma f (+ lo 1) hi))))
  (def (test w) (list (sigma (list (sym sq)) 1 w)
    (sigma (scale 2) 1 w)
    (sigma (linear 3 4) 1 w)))
  (def (sq x) (* x x))
  (def (scale c) (list (sym scale) c))
  (def (scaleHelper y c) (* c y))
  (def (linear a b) (list (sym linear) a b))
  (def (linearHelper z a b) (+ (* a z) b))
  (def (applyClosure clo arg)
    (bind name (nth 1 clo) ; Assume nth is 1-based list indexing
      (cond ((sym= name (sym sq)) (sq arg))
        ((sym= name (sym scale))
          (scaleHelper arg (nth 2 clo))
        ((sym= name (sym linear))
          (linearHelper arg (nth 2 clo) (nth 3 clo)))
        (else (error "unknown closure"))))))
Some languages with only top-level functions (particularly C) allow function values to be named, passed, returned, stored, but they cannot be created in any context (i.e., no closures).

Model this in FOFL-PLUS = FOFL + two constructs:

- \((\text{fref } F)\) returns the function value denoted by \(F\)
- \((\text{fapp } E_{rator} \ E_{rand_1} \ldots \ E_{rand_n})\) invokes the function denoted by \(E_{rator}\) to the values denoted by the operands \(E_{rand_1} \ldots E_{rand_n}\).

**FOFL-PLUS Example**

```plaintext
(fofl-plus (n) (list (app5 (fref inc)))
  (app5 (fref dbl))
  (app5 (fref mul-n)))

(def (inc x) (+ x 1))
(def (dbl y) (* y 2))
(def (mul-n z) (* z n))
(def (app5 f) (fapp f 5)))
```
Closure-converting $\sigma$ in FOFL-PLUS

(fofl-plus (n) (test n)
 (def (sigma f lo hi)
   (if (> lo hi) 0
       (+ (applyClosure f lo)
           (sigma f (+ lo 1) hi)))
 (def (test w)
   (list (sigma (list (frefsq)) 1 w)
     (sigma (scale 2) 1 w)
     (sigma (linear 3 4) 1 w)))
 (def (sq x clo) (* x x))
 (def (scale c) (list (fref scaleHelper) c))
 (def (scaleHelper y clo) (* (nth 2 clo) y))
 (def (linearHelper a b) (list (fref linearHelper) a b))
 (def (linearHelper z clo) (+ (* (nth 2 clo) z) (nth 3 clo)))
 (def (applyClosure clo arg) (fapp (nth 1 clo) arg clo))

Closure-converting $\sigma$ in JAVA, Part 1

interface IntFun {public int apply (int x);}

public static int sigma (int lo, int hi, IntFun f) {
  int sum = 0;
  for (int i = lo; i <= hi; i++) {
    sum = sum + f.apply(i);
  }
  return sum;
}

public static void main (String [] args) {
  int n = Integer.parseInt(args[0]);
  System.out.println(sigma(1, n, sqFun()));
  System.out.println(sigma(1, n, scaleFun(2)));
  System.out.println(sigma(1, n, linearFun(3,4)));
}
Closure-converting $\sigma$ in JAVA, Part 2

class Sigma {
    :
    public static IntFun sqFun () {return new SqFun ();}
    public static IntFun scaleFun (int c) {return new ScaleFun (c);}
    public static IntFun linearFun (int a, int b)
        {return new LinearFun (a,b);}}

class SqFun implements IntFun {public int apply (int x) {return x * x;}}

class ScaleFun implements IntFun {
    private int c;
    public ScaleFun (int c) {this.c = c;}
    public int apply (int y) {return c * y;}}

class LinearFun implements IntFun {
    private int a,b;
    public LinearFun (int a, int b) {this.a = a; this.b = b;}
    public int apply (int z) {return (a * z) + b;}}

Closure-converting $\sigma$ in JAVA, Part 2’

Can instead use invocations of *anonymous inner classes*:

public static IntFun sqFun () {
    return new IntFun () {public int apply (int x) {return x * x;}};
}

public static IntFun scaleFun (final int c) {
    return new IntFun () {public int apply (int x) {return c * x;}};
}

public static IntFun linearFun (final int a, final int b) {
    return new IntFun () {public int apply (int x) {return (a * x) + b;}};
}