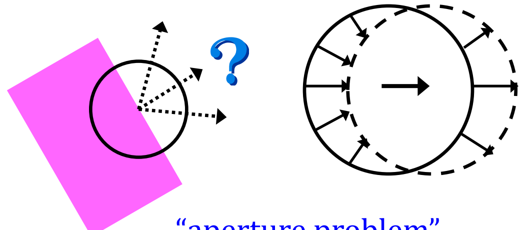
Analysis of Motion

Computing the velocity field

CS332 Visual Processing
Department of Computer Science
Wellesley College

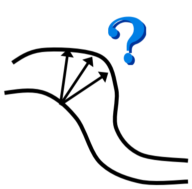
Measuring image motion

velocity field



"aperture problem"

"local" motion detectors only measure *component of motion* perpendicular to moving edge



2D velocity field not determined *uniquely* from the changing image

need *additional constraint* to compute a unique velocity field

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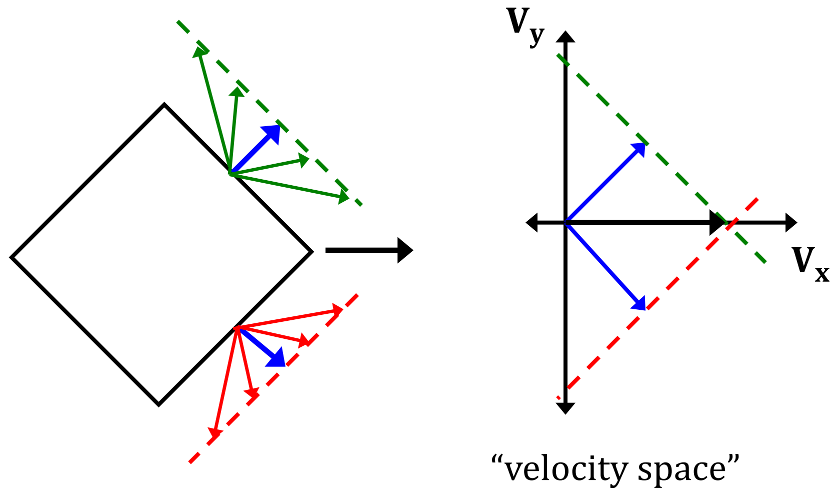
Option 1: Assume *pure translation*



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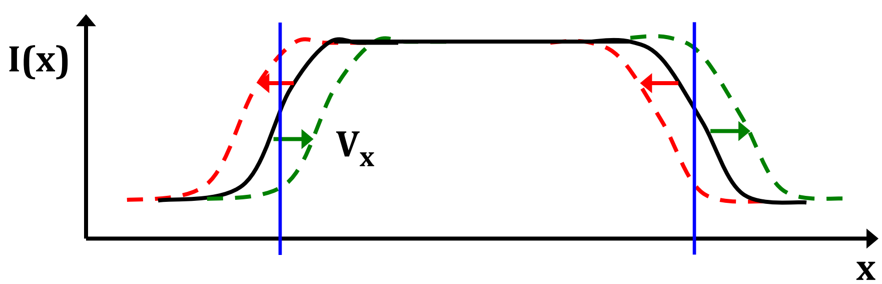
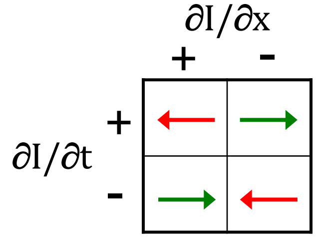
Measuring motion in one dimension



V_x = velocity in x direction

- rightward movement: $V_x > 0$
- leftward movement: $V_x < 0$
- speed: $| V_x |$
- pixels/time step

$$V_x = -\frac{\partial I/\partial t}{\partial I/\partial x}$$



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Measuring motion components in 2-D

(1) *gradient* of image intensity

$\nabla I = (\partial I/\partial x, \partial I/\partial y)$

(2) time derivative

$\partial I/\partial t$

(3) velocity along gradient:

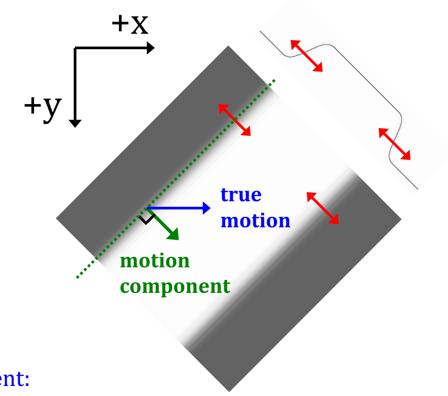
$v^{\perp}$

- movement in direction of gradient:

  $\mathbf{v^{\perp} > 0}$

- movement opposite direction of gradient:

  $\mathbf{v^{\perp} < 0}$



$$\mathbf{v}^{\perp} = -\frac{\partial I/\partial t}{|\nabla I|} = -\frac{\partial I/\partial t}{[(\partial I/\partial x)^2 + (\partial I/\partial y)^2]^{1/2}}$$

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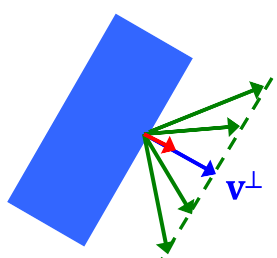
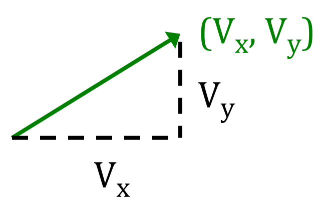
2-D velocities (V_x, V_y) consistent with $v^{\perp}$





All (V_x, V_y) such that the component of (V_x, V_y) in the direction of the gradient is $\mathbf{v}^{\perp}$

(u_x, u_y): unit vector in direction of gradient

Use the *dot product:* $(V_x, V_y) \bullet (u_x, u_y) = \mathbf{v}^{\perp}$

$$V_x u_x + V_y u_y = \mathbf{v}^{\perp}$$

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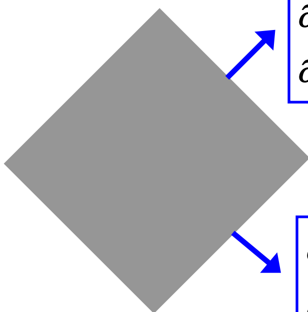
Exercise …



$\partial I/\partial x = 10$

$\partial I/\partial y = -10$

$\partial I/\partial t = -30$

$\rightarrow (V_x, V_y)$??

$\partial I/\partial x = 10$

$\partial I/\partial y = 10$

$\partial I/\partial t = -30$

For each component:

(1) u_x

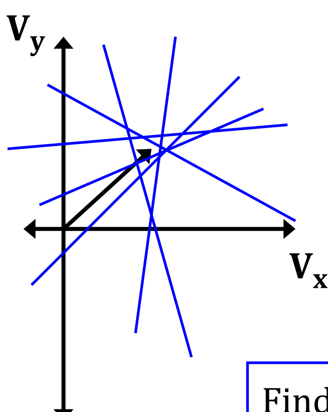
(2) u_y

(3) $v^{\perp}$

(4) $V_x u_x + V_y u_y = v^{\perp}$

solve for V_x, V_y

In practice…



Previously…

$V_x u_x + V_y u_y = v^{\perp}$

New strategy:

Find (V_x, V_y) that ***best fits*** all motion components together

Find (V_x, V_y) that minimizes:

$$\sum(V_x u_x + V_y u_y - v^{\perp})^2$$

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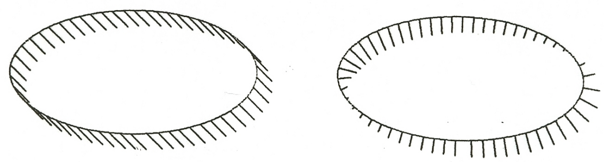
Option 2: *Smoothness* assumption:

Compute a velocity field that:

(1) is consistent with local measurements of image motion (perpendicular components)

(2) has the *least amount of variation* possible

Pure Translation:



true & smoothest velocity field

initial motion measurements

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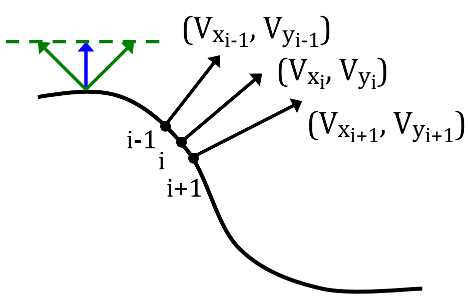
Computing the smoothest velocity field



motion components:

$V_{x_i} u_{x_i} + V_{y_i} u_{y_i} = v^{\perp}_i$

change in velocity:

$(V_{x_{i+1}} - V_{x_i}, V_{y_{i+1}} - V_{y_i})$

Find (V_{x_i}, V_{y_i}) that minimize:

$$\Sigma (V_{x_i} u_{x_i} + V_{y_i} u_{y_i} - v^{\perp}_i)^2 + \lambda [(V_{x_{i+1}} - V_{x_i})^2 + (V_{y_{i+1}} - V_{y_i})^2]$$

deviation from image motion measurements **+** variation in velocity field

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