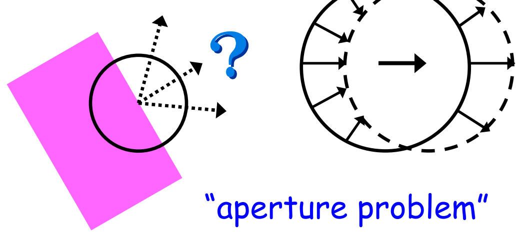
Analysis of Motion

Computing the velocity field

CS332 Visual Processing
Department of Computer Science
Wellesley College

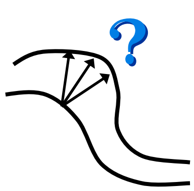
Measuring image motion

velocity field



"aperture problem"

"local" motion detectors only measure *component of motion* perpendicular to moving edge

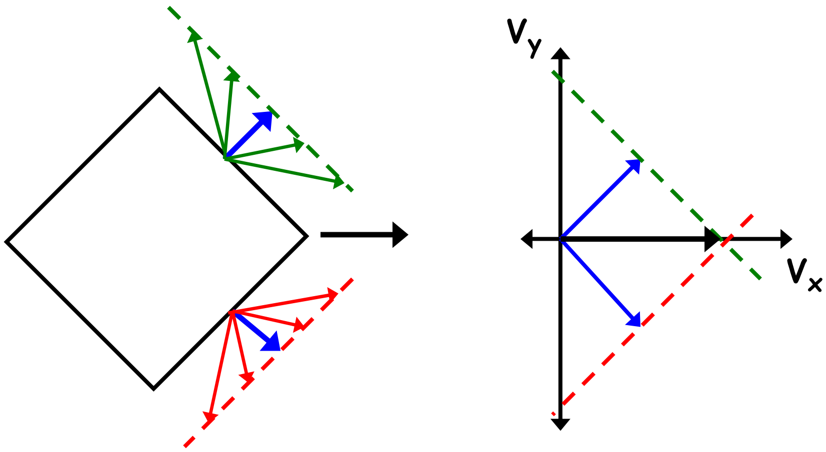


2D velocity field not determined uniquely from the changing image

need *additional constraint* to compute a unique velocity field

1-2

Assume *pure translation* or *constant velocity*



In practice…

- Error in initial motion measurements
- Velocities not constant locally
- Image features with small range of orientations

1-3

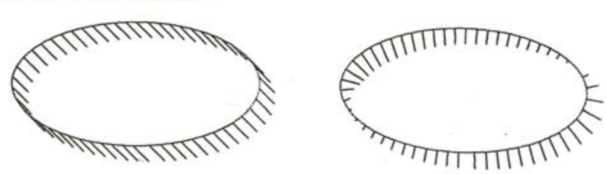
Smoothness assumption:

Compute a velocity field that:

(1) is consistent with local measurements of image motion (perpendicular components)

(2) has the *least amount of variation* possible

Pure Translation:

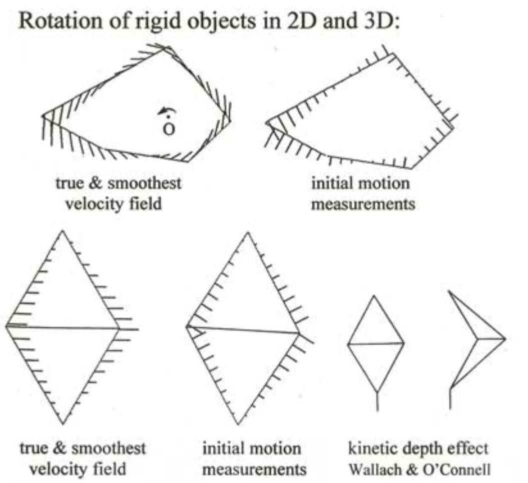


true & smoothest velocity field

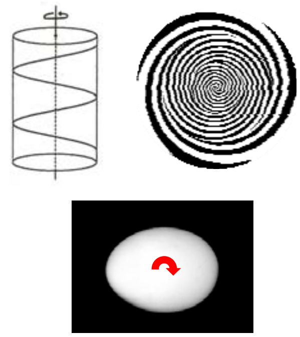
initial motion measurements

1-4

When is the smoothest velocity field *correct?*



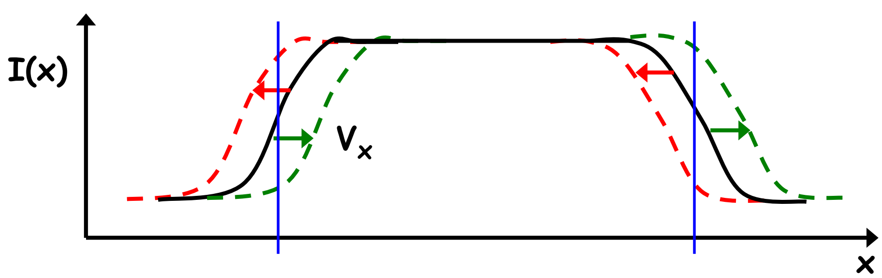
When is it *wrong?*



motion *illusions*

1-5

Measuring motion in one dimension



V_x = velocity in x direction

- rightward movement: $V_x > 0$
- leftward movement: $V_x < 0$
- speed: $|V_x|$
- pixels/time step

$$V_x = -\frac{\partial I/\partial t}{\partial I/\partial x}$$

	$\partial I/\partial x$ +	$\partial I/\partial x$ –
$\partial I/\partial t$ +		
$\partial I/\partial t$ –		

1-6

Measurement of motion components in 2-D

(1) *gradient* of image intensity

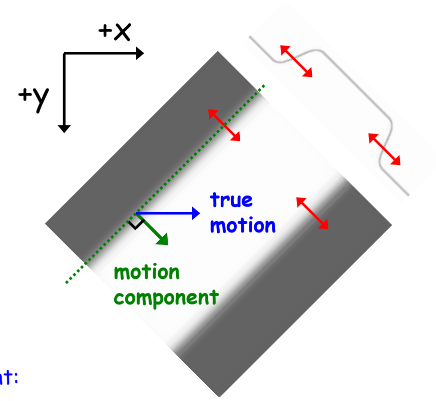
$\nabla I = (\partial I/\partial x, \partial I/\partial y)$

(2) time derivative

$\partial I/\partial t$

(3) velocity along gradient:

$v^{\perp}$



- movement in direction of gradient:

  $v^{\perp} > 0$

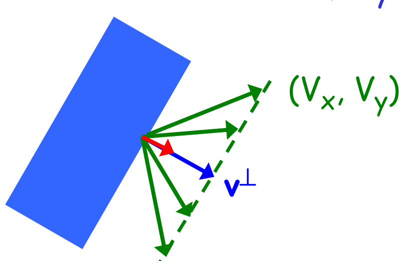
- movement opposite direction of gradient:

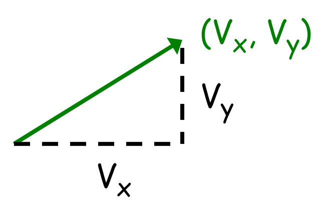
  $v^{\perp} < 0$

$$v^{\perp} = -\frac{\partial I/\partial t}{|\nabla I|} = -\frac{\partial I/\partial t}{[(\partial I/\partial x)^2 + (\partial I/\partial y)^2]^{1/2}}$$

1-7

2-D velocities (V_x, V_y) consistent with $v^{\perp}$





All (V_x, V_y) such that the component of (V_x, V_y) in the direction of the gradient is $\mathbf{v}^{\perp}$

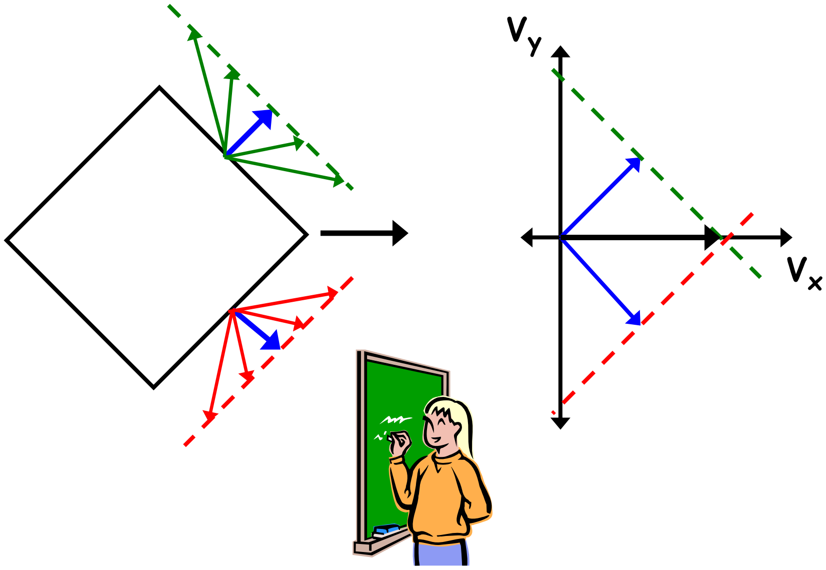
(u_x, u_y): unit vector in direction of gradient

Use the *dot product:* $(V_x, V_y) \bullet (u_x, u_y) = \mathbf{v}^{\perp}$

$$V_x u_x + V_y u_y = \mathbf{v}^{\perp}$$

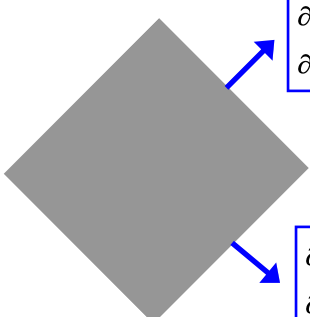
1-8

Time-out exercise



1-9

Details…



$\partial I/\partial x = 10$
$\partial I/\partial y = -10$
$\partial I/\partial t = -30$

(Vx, Vy) ??

$\partial I/\partial x = 10$
$\partial I/\partial y = 10$
$\partial I/\partial t = -30$

For each component:

(1) u_x

(2) u_y

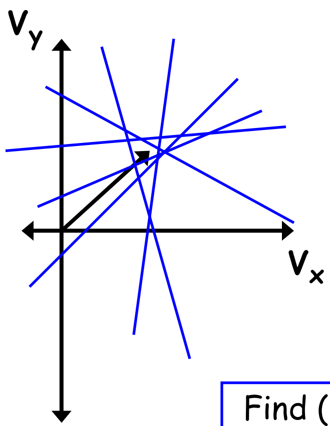
(3) $v^{\perp}$

(4) $V_x u_x + V_y u_y = v^{\perp}$

solve for V_x, V_y

1-10

In practice…



Previously…

$V_x u_x + V_y u_y = v^{\perp}$

New strategy:

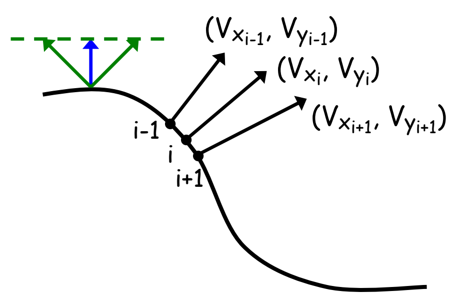
Find (V_x, V_y) that ***best fits*** all motion components together

Find (Vx, Vy) that minimizes:

$$\sum(V_x u_x + V_y u_y - v^{\perp})^2$$

1-11

Computing the smoothest velocity field



motion components:

$V_{x_i} u_{x_i} + V_{y_i} u_{y_i} = v^{\perp}_i$

change in velocity:

$(V_{x_{i+1}} - V_{x_i}, V_{y_{i+1}} - V_{y_i})$

Find (V_{x_i}, V_{y_i}) that minimize:

$$\sum(V_{x_i} u_{x_i} + V_{y_i} u_{y_i} - v^{\perp}_i)^2 + \lambda[(V_{x_{i+1}} - V_{x_i})^2 + (V_{y_{i+1}} - V_{y_i})^2]$$

1-12