Analysis of Motion

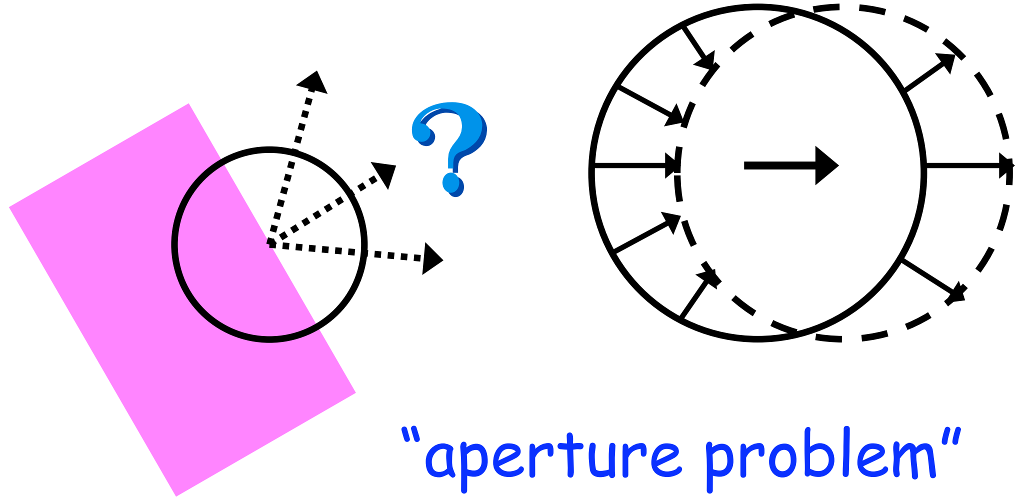
Motion Review

CS332 Visual Processing

Department of Computer Science
Wellesley College

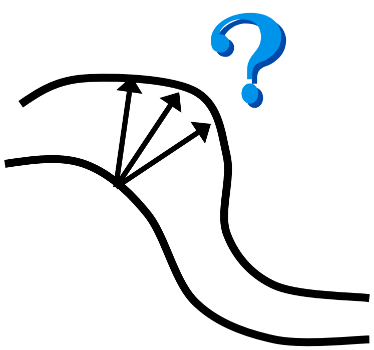
Measuring image motion

velocity field



"aperture problem"

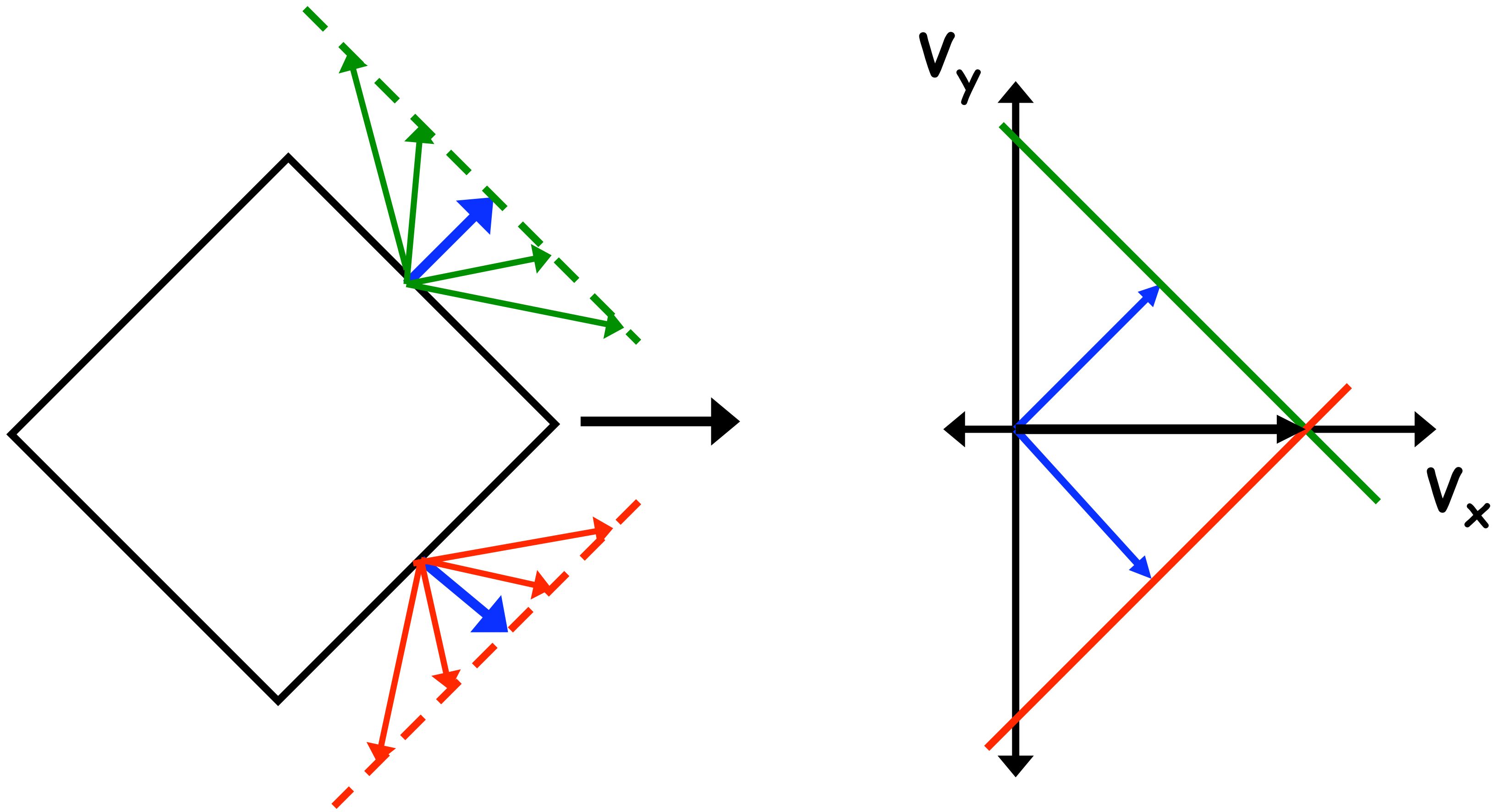
"local" motion detectors only measure *component of motion* perpendicular to moving edge



2D velocity field not determined uniquely from the changing image

need *additional constraint* to compute a unique velocity field

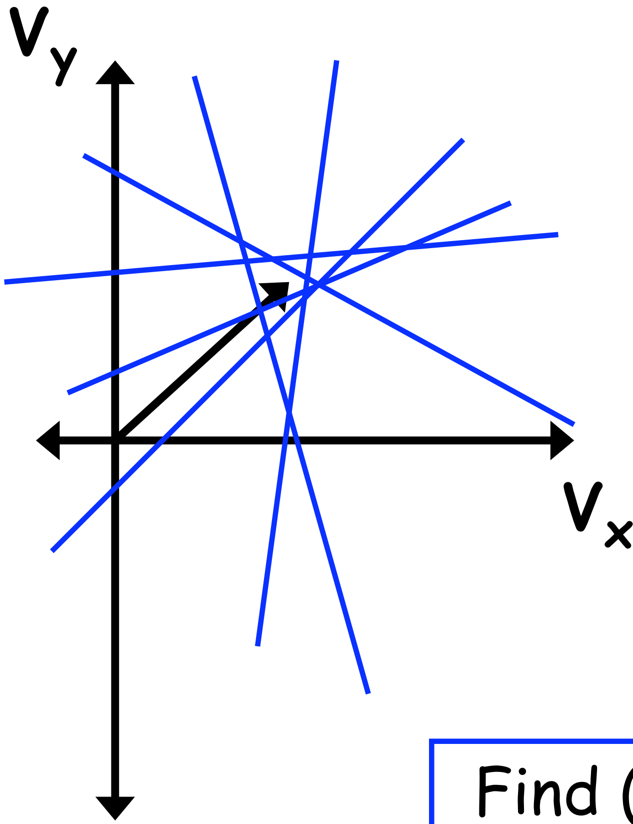
Assume pure translation or *constant velocity*



In practice...

- Error in initial motion measurements
- Velocities not constant locally
- Image features with small range of orientations

In practice…



Previously…

$V_x u_x + V_y u_y = v^{\perp}$

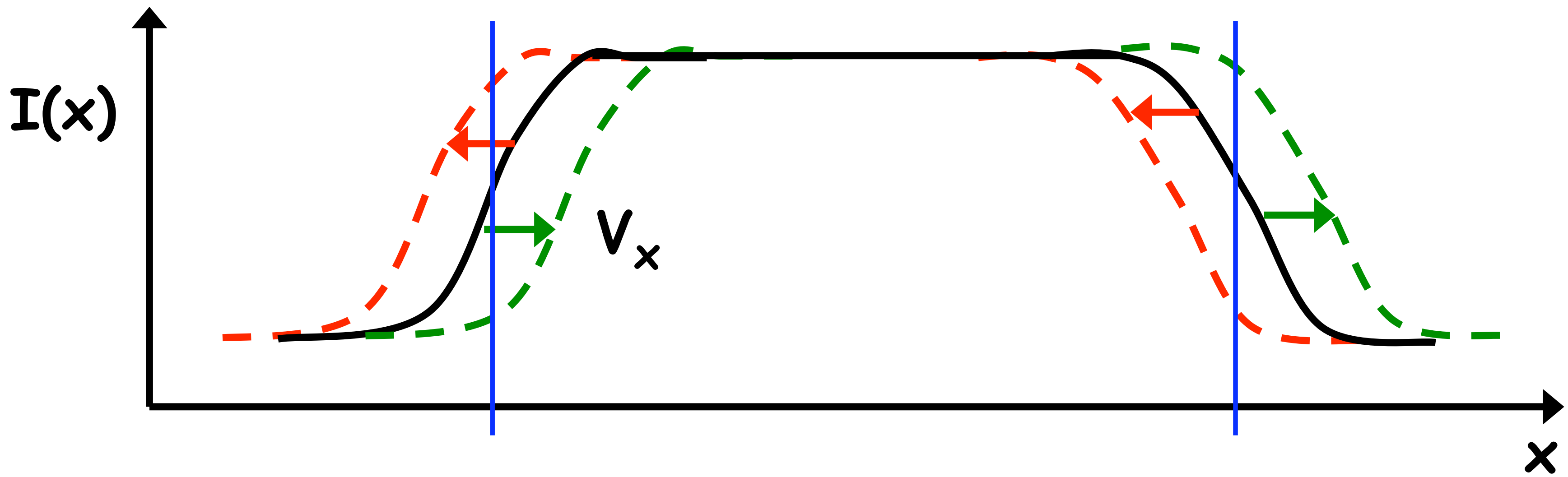
New strategy:

Find (V_x, V_y) that ***best fits*** all motion components together

Find (Vx, Vy) that minimizes:

$$\sum (V_x u_x + V_y u_y - v^{\perp})^2$$

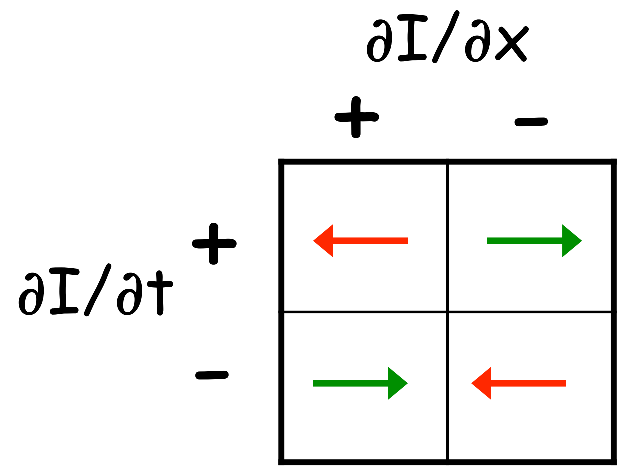
Measuring motion in one dimension



V_x = velocity in x direction

- rightward movement: $V_x > 0$
- leftward movement: $V_x < 0$
- speed: $|V_x|$
- pixels/time step

$$V_x = -\frac{\partial I/\partial t}{\partial I/\partial x}$$



Measurement of motion components in 2-D

(1) *gradient* of image intensity

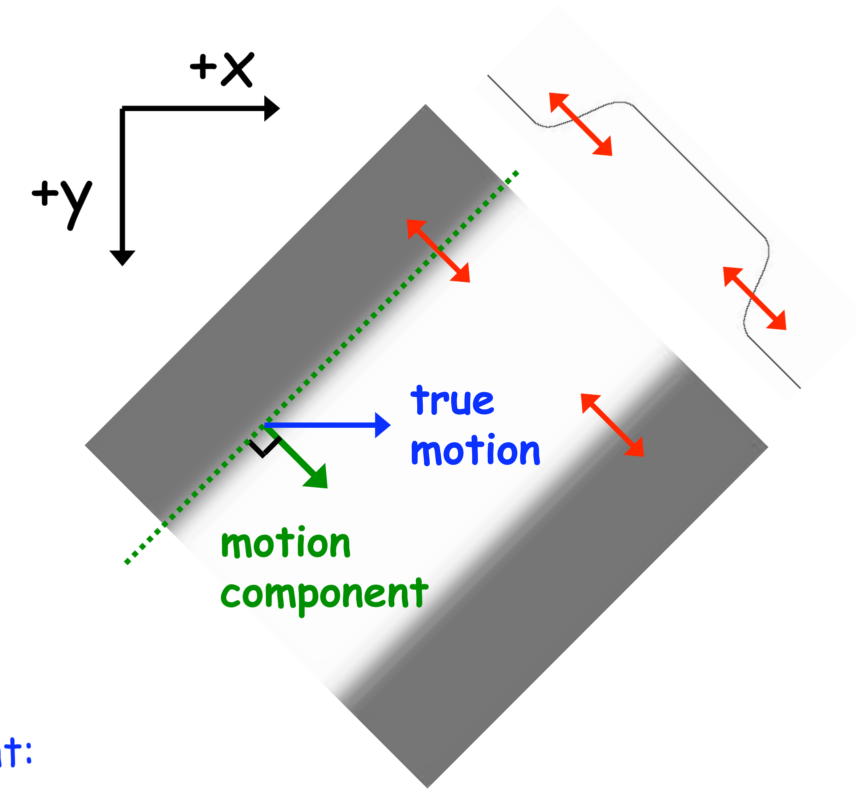
$$\nabla I = (\partial I/\partial x, \partial I/\partial y)$$

(2) time derivative

$$\partial I/\partial t$$

(3) velocity along gradient:

$$v^{\perp}$$



- movement in direction of gradient:

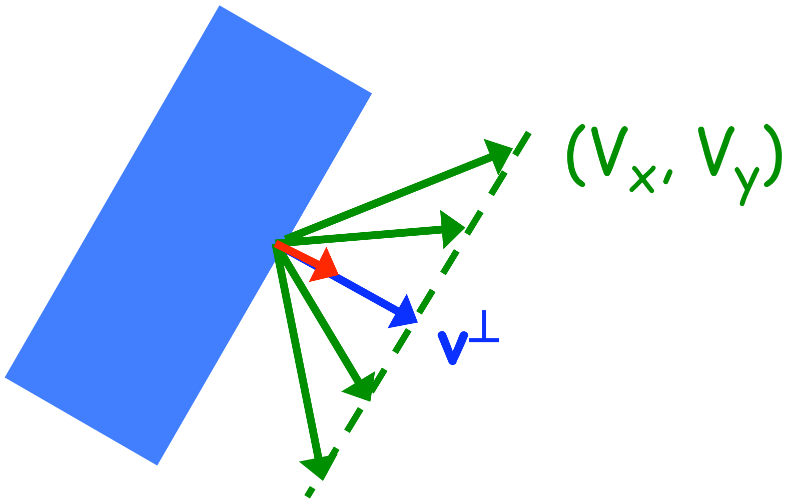
  $v^{\perp} > 0$

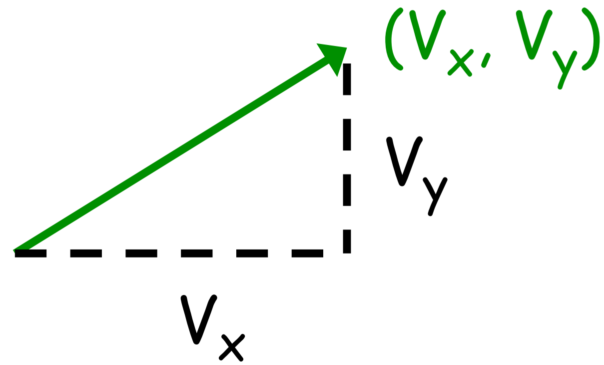
- movement opposite direction of gradient:

  $v^{\perp} < 0$

$$v^{\perp} = -\frac{\partial I/\partial t}{|\nabla I|} = -\frac{\partial I/\partial t}{[(\partial I/\partial x)^2 + (\partial I/\partial y)^2]^{1/2}}$$

2-D velocities (V_x,V_y) consistent with $v^{\perp}$





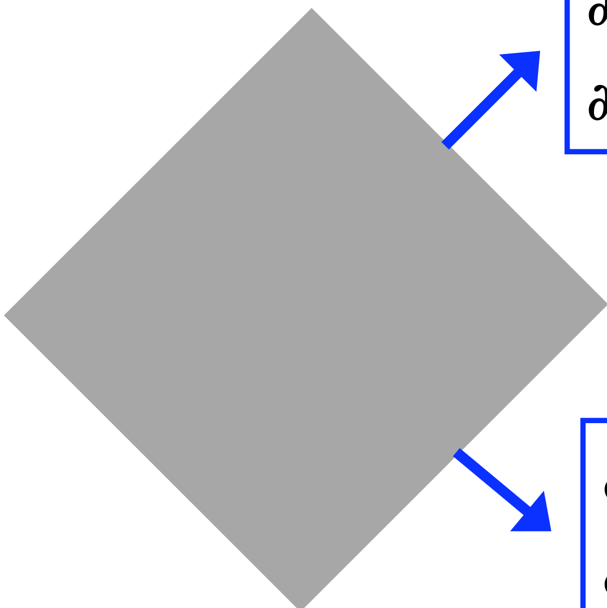
All (V_x, V_y) such that the component of (V_x, V_y) in the direction of the gradient is $\mathbf{v}^{\perp}$

(u_x, u_y): unit vector in direction of gradient

Use the *dot product:* $(V_x, V_y) \bullet (u_x, u_y) = \mathbf{v}^{\perp}$

$$V_x u_x + V_y u_y = \mathbf{v}^{\perp}$$

Details...



$\partial I/\partial x = 10$

$\partial I/\partial y = -10$

$\partial I/\partial t = -30$

→ (V_x, V_y) ??

$\partial I/\partial x = 10$

$\partial I/\partial y = 10$

$\partial I/\partial t = -30$

For each component:

(1) u_x

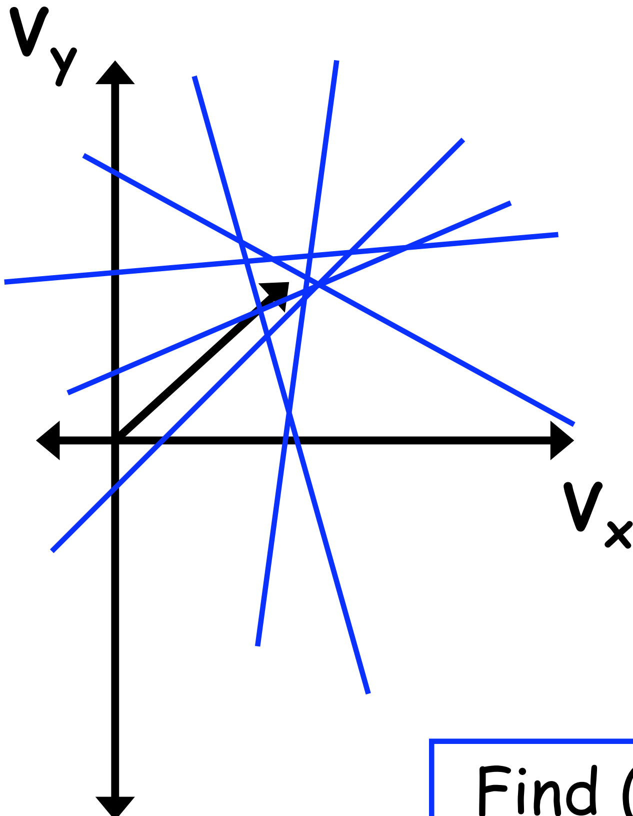
(2) u_y

(3) $v^{\perp}$

(4) $V_x u_x + V_y u_y = v^{\perp}$

solve for V_x, V_y

In practice…



Previously…

$$V_x u_x + V_y u_y = v^{\perp}$$

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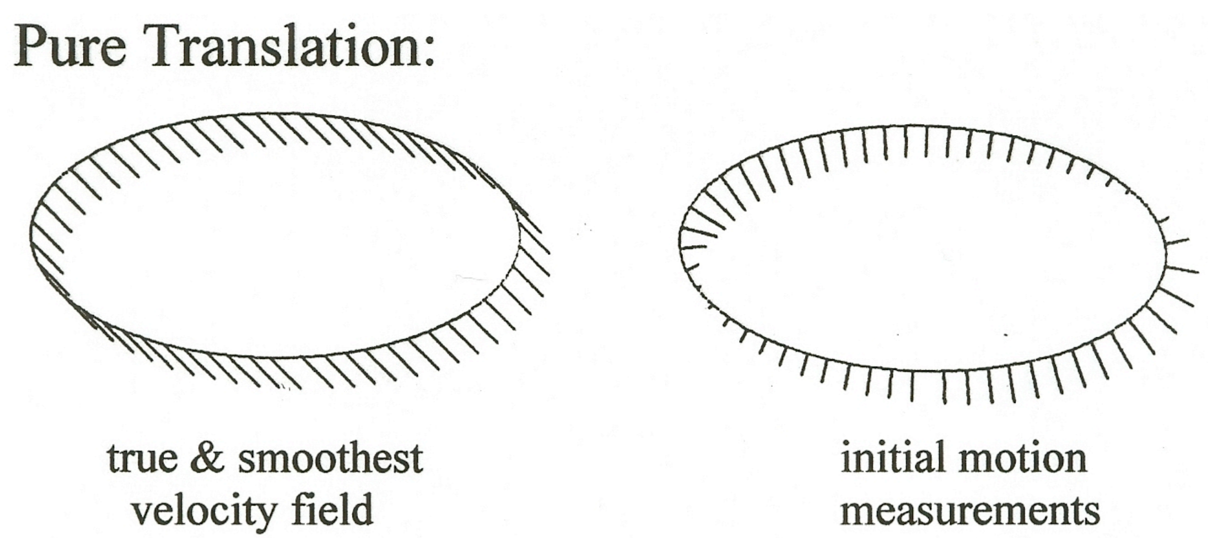
Smoothness assumption:

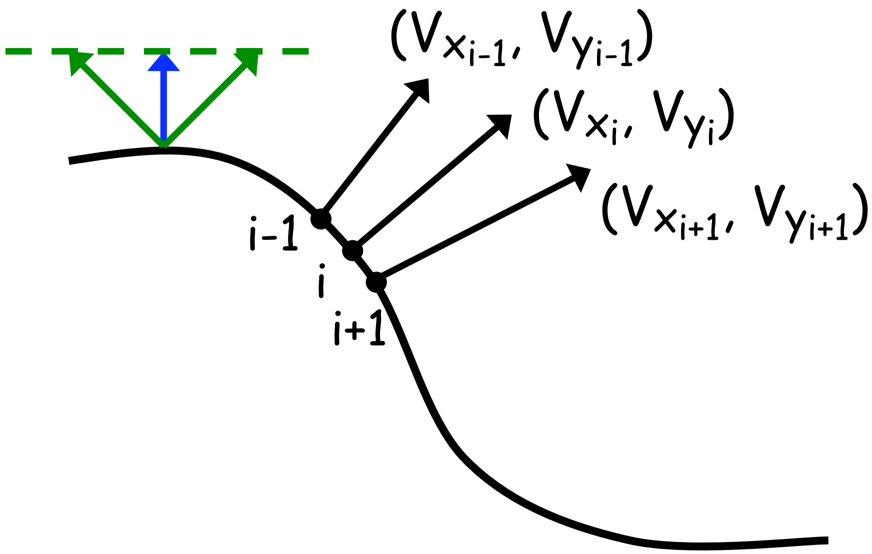
Compute a velocity field that:

(1) is consistent with local measurements of image motion (perpendicular components)

(2) has the *least amount of variation* possible



Computing the smoothest velocity field



motion components:

$$V_{x_i} u_{x_i} + V_{y_i} u_{y_i} = v^{\perp}{}_i$$

change in velocity:

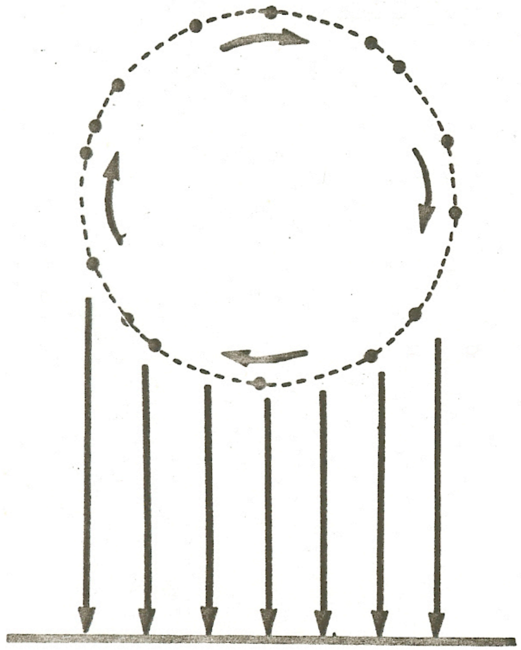
$$(V_{x_{i+1}} - V_{x_i}, V_{y_{i+1}} - V_{y_i})$$

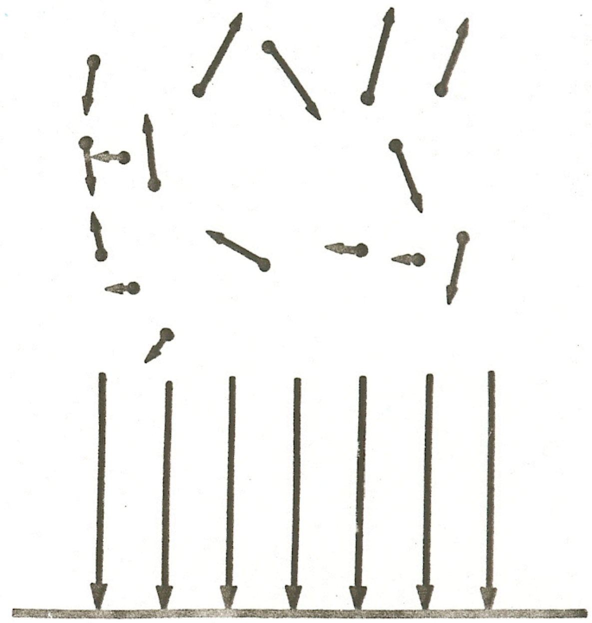
Find (V_{x_i}, V_{y_i}) that minimize:

$$\sum (V_{x_i} u_{x_i} + V_{y_i} u_{y_i} - v^{\perp}{}_i)^2 + \lambda[(V_{x_{i+1}} - V_{x_i})^2 + (V_{y_{i+1}} - V_{y_i})^2]$$

Ambiguity of 3D recovery

birds' eye views



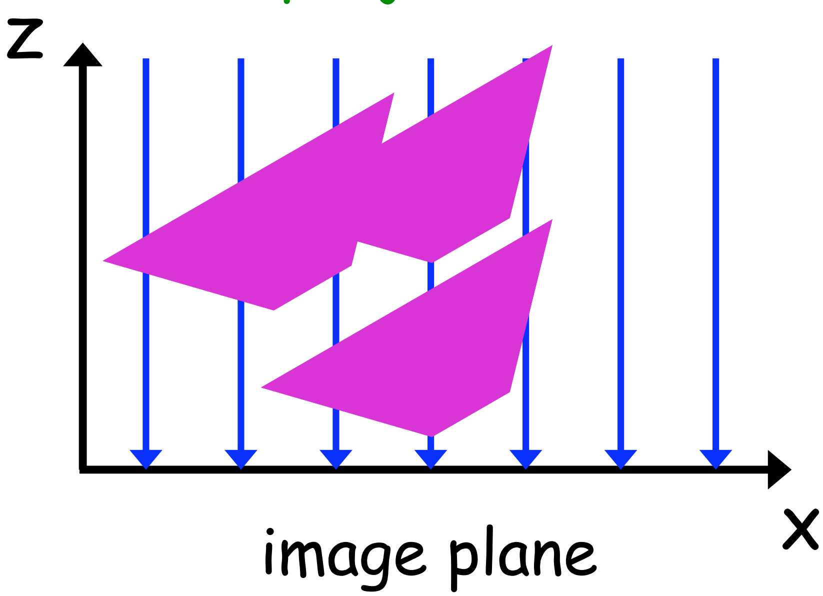


We need *additional constraint* to recover 3D structure uniquely

"rigidity constraint"

Image projections

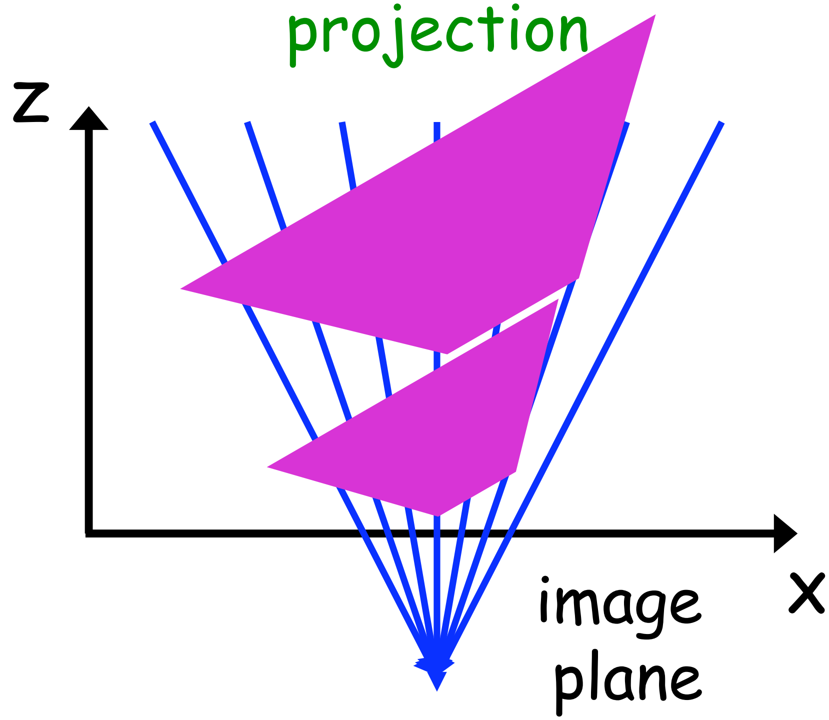
orthographic projection



$(X, Y, Z) \rightarrow (X, Y)$

- only relative depth
- requires object rotation

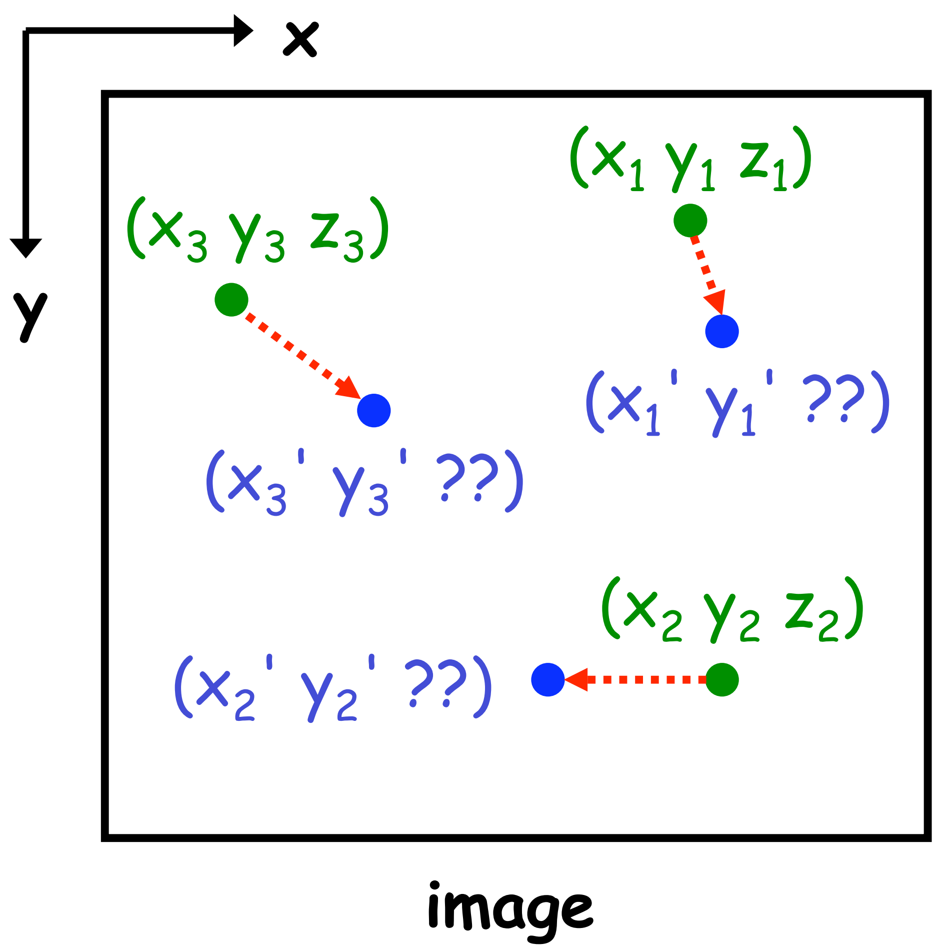
perspective projection



$(X, Y, Z) \rightarrow (X/Z, Y/Z)$

- only scaled depth
- requires translation of observer relative to scene

Incremental Rigidity Scheme



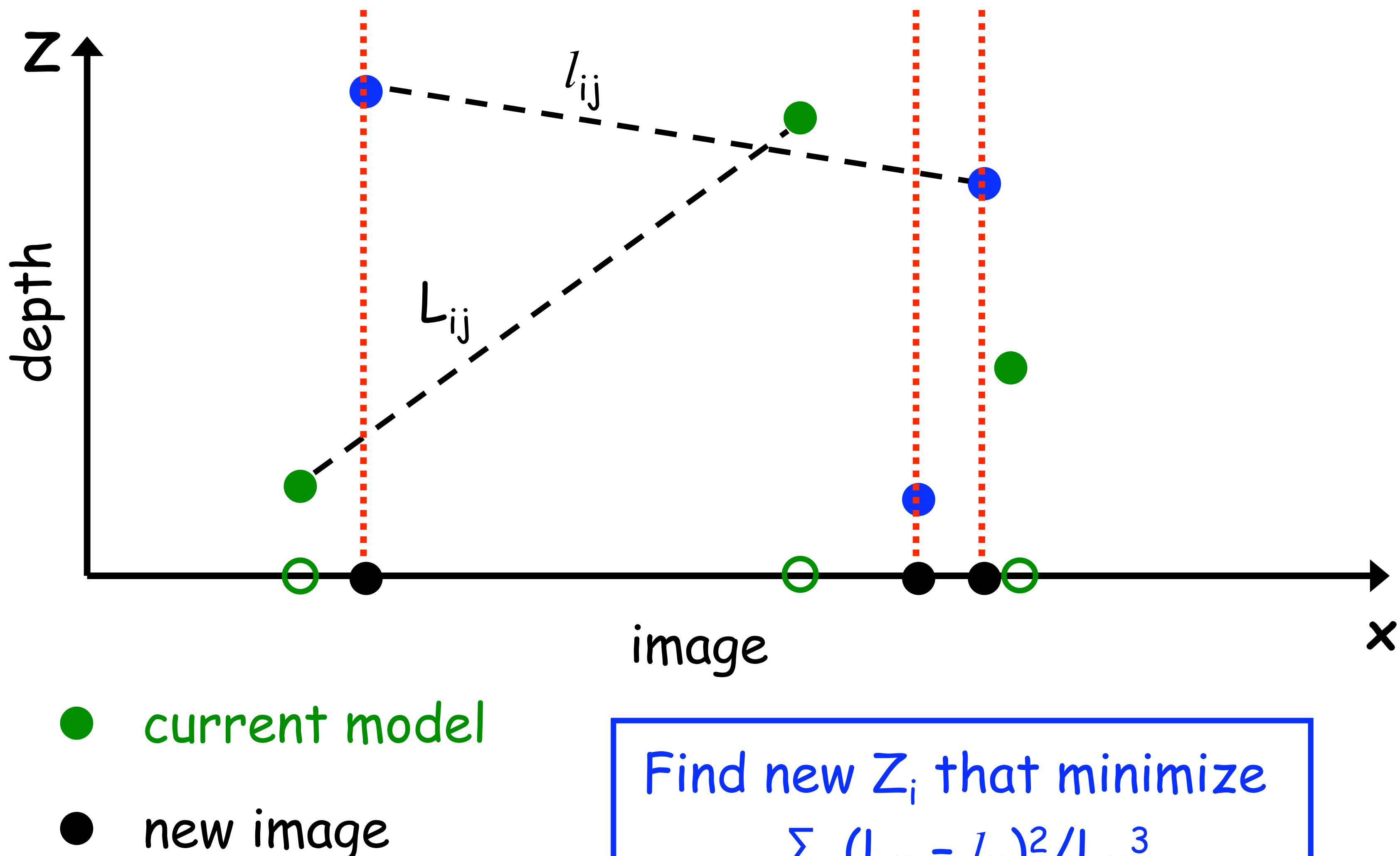
depth: Z

initially, Z=0
at all points

Find new 3D model that
maximizes rigidity

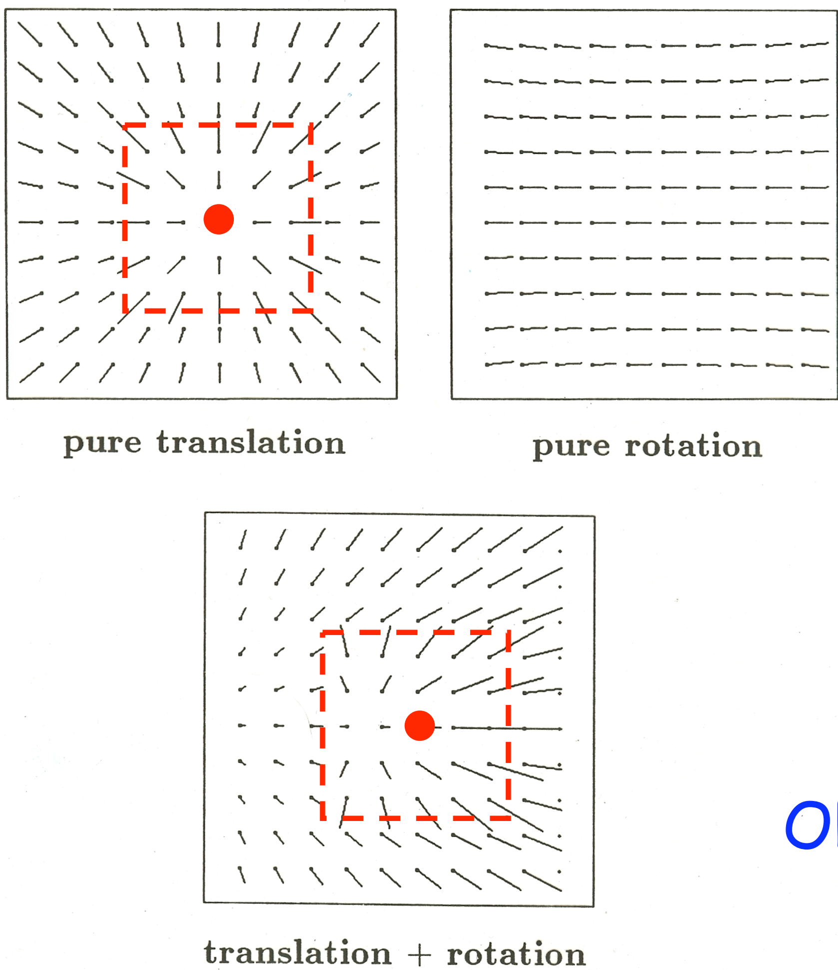
Compute new Z values
that minimize change
in 3D structure

Bird's eye view:



Find new Z_i that minimize
$\Sigma \ (L_{ij} - l_{ij})^2/L_{ij}^3$

Observer motion problem, revisited



From image motion, compute:

- Observer translation

  $(T_x \ T_y \ T_z)$

- Observer rotation

  $(R_x \ R_y \ R_z)$

- Depth at every location

  $Z(x,y)$

Observer undergoes both translation + rotation

Equations of observer motion

Translation	Rotation	Depth
(T_x, T_y, T_z)	(R_x, R_y, R_z)	$Z(x,y)$

$$V_x = (-T_x + xT_z)/Z + R_x xy - R_y(x^2+1) + R_z y$$

$$V_y = (-T_y + yT_z)/Z + R_x(y^2+1) - R_y xy - R_z x$$

Translational Component: $(-T_x + xT_z)/Z$, $(-T_y + yT_z)/Z$

Rotational Component: $R_x xy - R_y(x^2+1) + R_z y$, $R_x(y^2+1) - R_y xy - R_z x$

Translational component of velocity

$V_x = (-T_x + xT_z)/Z$

$V_y = (-T_y + yT_z)/Z$

Where is the FOE?

x = y =

Example 1: $T_x = T_y = 0$ $T_z = 1$ $Z = 10$ everywhere

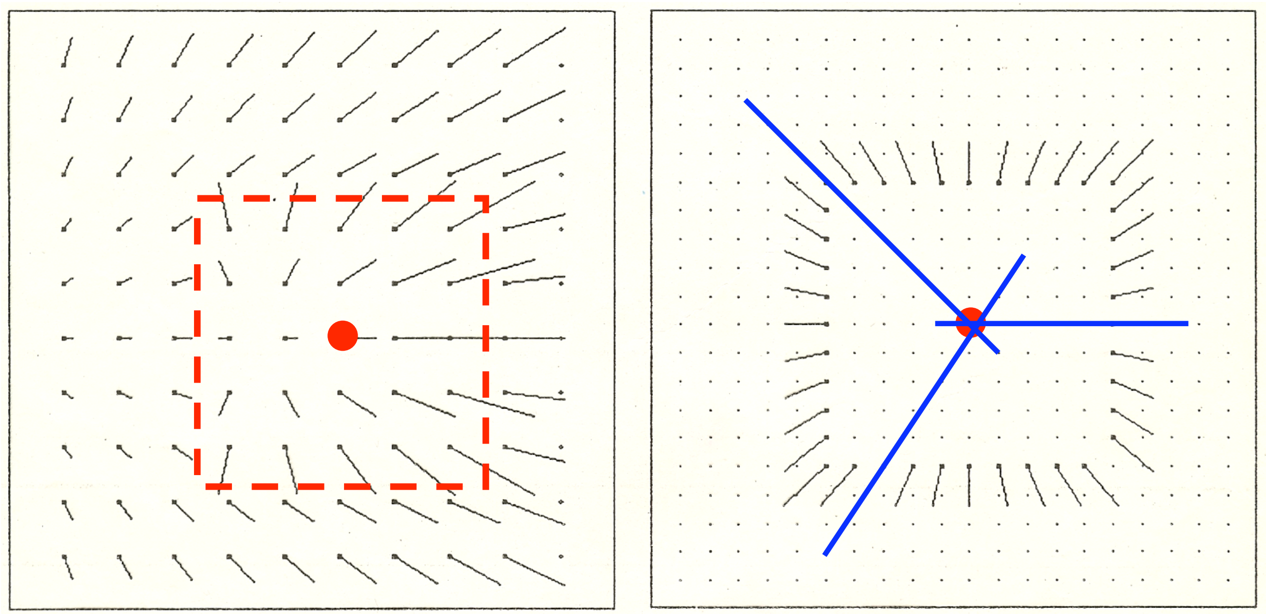
$V_x =$ ______ $V_y =$ ______

Sketch the velocity field

Example 2: $T_x = T_y = 2$ $T_z = 1$ $Z = 10$ everywhere

$V_x =$ ______ $V_y =$ ______

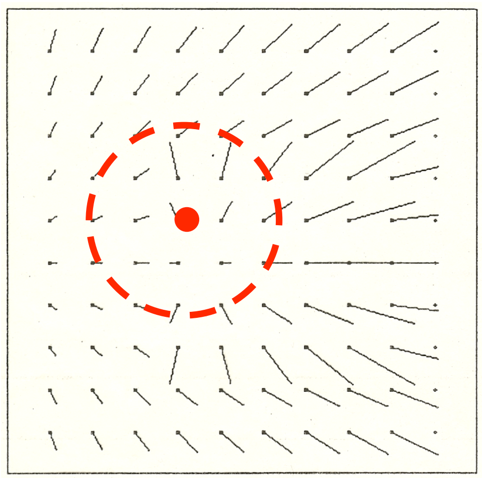
Longuet-Higgins & Prazdny



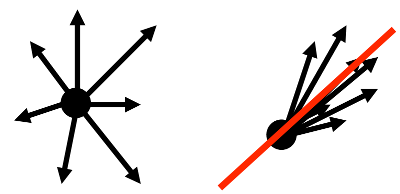
- *Along a depth discontinuity, velocity differences* depend only on observer translation
- Velocity differences point to the focus of expansion

Rieger & Lawton's algorithm

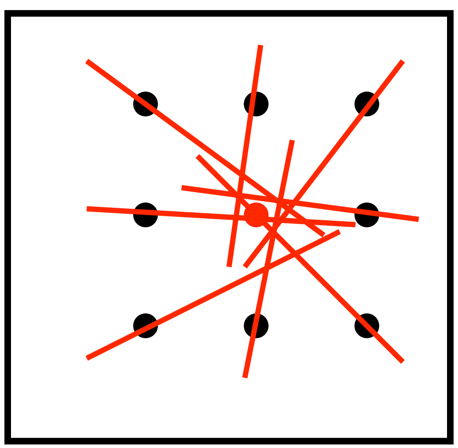
- At each image location, compute distribution of velocity differences within neighborhood



Appearance of sample distributions:



- Find points with strongly oriented distribution, compute dominant direction



- Compute focus of expansion from intersection of dominant directions