Lab Modulation Solutions

October 20, 2021

Handwritten Exercises

1. Ring and Amplitude Modulation

   (a) This is ring modulation. From the slides, the partials produced by this expression can be expressed as $\frac{A_c A_m}{2}[\sin(2\pi(f_c - f_m)t + \frac{\pi}{2}) + \sin(2\pi(f_c + f_m)t - \frac{\pi}{2})]$. Therefore, the partials produced are 100Hz and 300Hz both at amplitudes of 0.25 and phases of $\pi/2$ and $-\pi/2$, respectively.

   (b) This is amplitude modulation. From the slides, the partials produced by this expression can be expressed as $\frac{A_c A_m}{2}[\sin(2\pi(f_c - f_m)t + \frac{\pi}{2}) + \sin(2\pi(f_c + f_m)t - \frac{\pi}{2})] + A_c \sin(2\pi f_c t)$. Therefore, the partials produced are the same as before plus a frequency of 100Hz at amplitude of 1 with phase 0.

2. In amplitude modulation, the modulation index controls the amplitude of the sidebands. A higher modulation index leads to a higher amplitude. A modulation index of zero produces only the carrier wave with no side bands.

3. Frequency Modulation

   (a) This is frequency modulation. The carrier wave here is 100Hz with a modulating frequency of 100Hz. The frequencies produced will be $100 \pm 100n$ where n is an integer. Because negative frequencies are simply positive frequencies with a phase shift, it also acceptable to thing about this as $100 + 100n$ where n is a natural number.

   (b) This is frequency modulation. The carrier wave here is 200Hz with a modulating frequency of 300Hz. The frequencies produced will be $200 \pm 300n$ where n is an integer. Because negative frequencies are simply positive frequencies with a phase shift, it also acceptable to thing about this as the following sequence of frequencies: 100, 200, 400, 500, 700, 800, etc.

4. In frequency modulation, the modulation index controls the amplitude of the sidebands. A higher modulation index leads to a higher amplitude. A modulation index of zero produces only the carrier wave no side bands. The modulation index for both frequency and amplitude modulation work in exactly the same way.

5. The expression $\sin(2\pi(100)t + 20\sin(2\pi(100)t))$ is simply frequency modulation producing the partials stated in 3b. Call that signal $x(t)$. Then ring modulation is applied to $x(t)$ with the signal $\sin(2\pi(50)t)$, meaning that every frequency in $x(t)$ produces additional sidebands. For example, 100Hz in $x(t)$ now produces 50Hz and 150Hz because it is multiplied by a frequency of 50Hz. 200Hz in $x(t)$ now produces 150Hz and 250Hz because it is multiplied by a frequency of 50Hz. So on and so forth. Thus, the partials produced are $50 + 100n$ where n is a natural number.