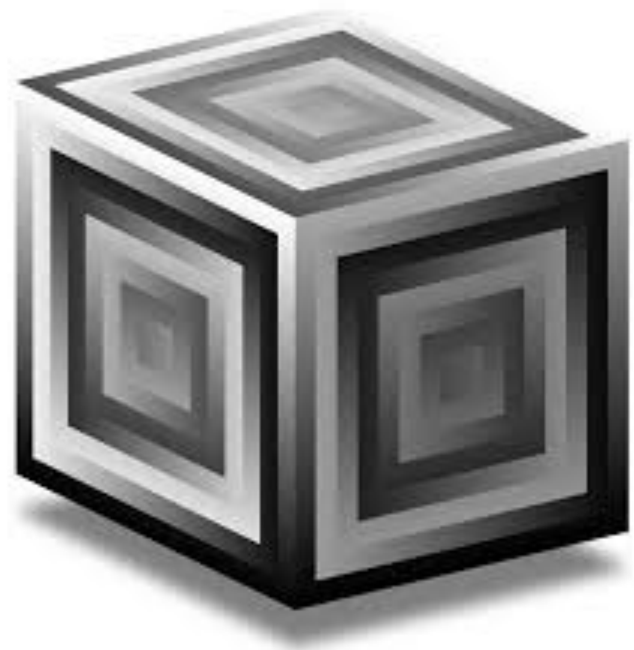
Amplitude Multiplication

Topics Addressed

- Envelope Review
- Multiplying Sine Waves
- Ring Modulation
- Sidebands
- Amplitude Modulation
- Modulation Index
- AM Radio

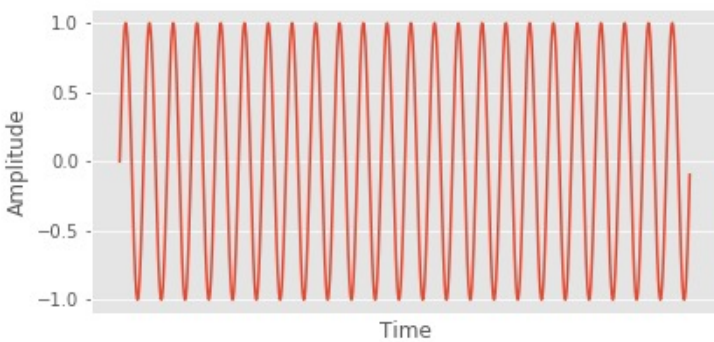


Multiplying Signals

- We have seen how multiplying an envelope and a signal can shape the amplitude of a sound. Generalizing this concept, multiplying the amplitudes of any two waves can produce a variety of effects that are not constrained to electronic music.
  - Ring Modulation
  - Amplitude Modulation
- AM/FM Radio stands for amplitude modulation and frequency modulation. They are the essential foundations for radio technology. Amplitude Modulation (which is based on multiplying two signals) is one of the main uses of signal multiplication.
  - Note that amplitude modulation in the context of radio has specific meaning. The two signals are broken down into what are called the carrier signal (the frequency your radio is tuned to) and the modulating signal (i.e., the song or voice you hear on the radio). These two terms can also be applied in a musical context. We can think of our main signal as the carrier and the envelope as the modulating signal.
- Amplitude modulation is also a foundation of electronic music. It can be used to produce a variety of effects.
- Note that the terms amplitude multiplication and amplitude modulation can be confounded. Amplitude multiplication is an umbrella term to refer to any form of multiplication with signals. Amplitude modulation is a form of signal multiplication but has a specific mathematical definition which we discuss in the forthcoming slides.

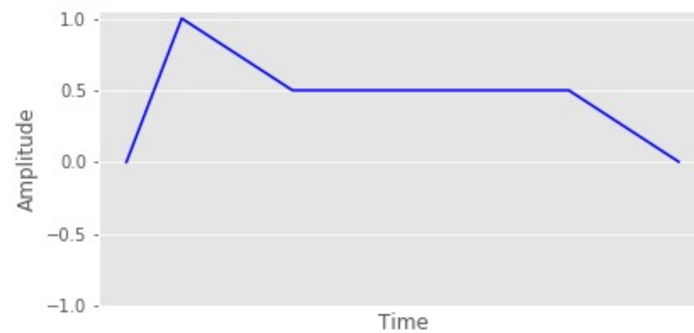
Envelopes

Signal

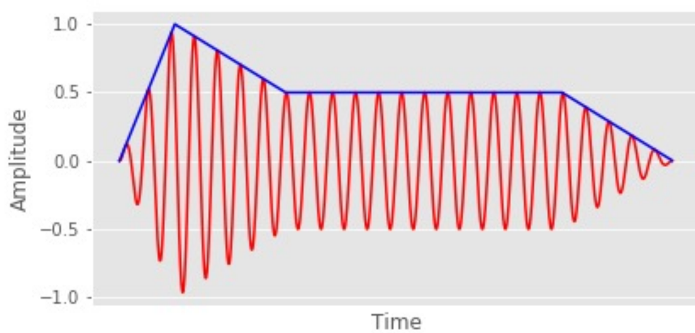


*

Envelope



=



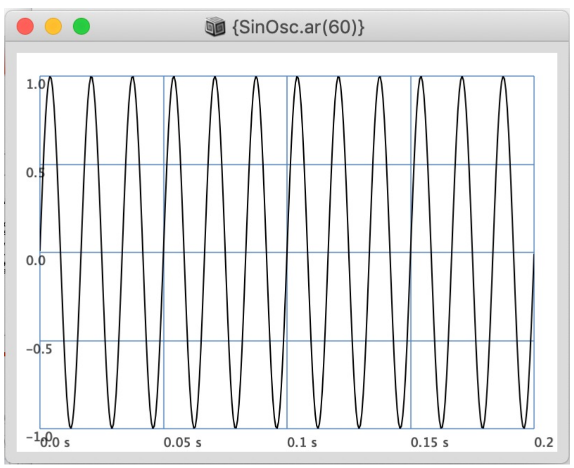
This form of amplitude multiplication uses a slow developing signal like an envelope to shape the signal on the left. We don’t actually perceive the envelope itself except in the sense that we hear its shape reflected in the volume of the original signal.

Multiplying Sine Waves

- Let's consider what happens when we modulate two simple sine waves.
- We will have two sine waves: the carrier wave and the modulating wave. In general, the carrier wave and the modulating wave could be of any frequency, but for the purposes of this introduction let's make the carrier wave's frequency much higher than the modulating wave.
- It is common to extend the notion of carrier signal and modulating signal from the realm of radio to digital music.

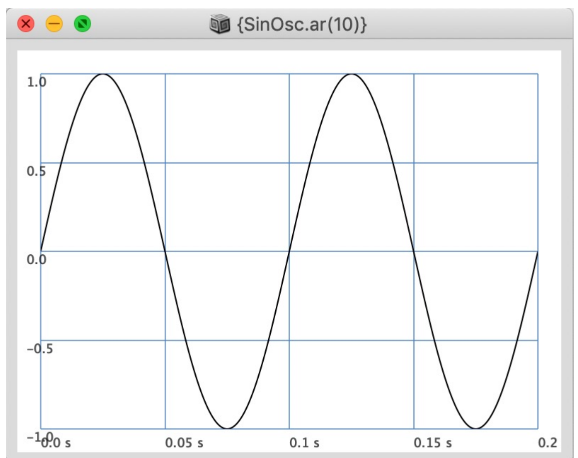
Multiplying Sine Waves

- The modulating frequency of 10Hz is below 20Hz (sub-audio range) so we perceive the resulting frequency as a quick succession of swells of the carrier frequency. Frequencies below 20Hz are called Low Frequency Oscillators (LFOs).
- When the modulating frequency is an LFO, this swelling effect is sometimes called the tremolo effect. Note that this is **not** the same thing as vibrato which is slight variations in pitch.



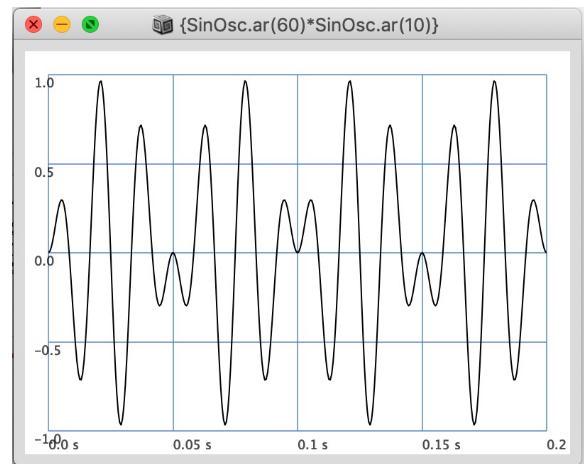
Carrier Frequency

*



Modulating Frequency

=



Result

Modulating Frequency

- Notice that when the modulating frequency starts to exceed about 1Hz, the sound seems to be out of tune. It is still fused as one sound, but some issue seems to be happening with intonation.
- What is contributing to that detuning? How can we explain it?
- What happens when the modulating frequency is in the audio range?

```
{SinOsc.ar(400)*SinOsc.ar(1)!2}.play;
```

Carrier Frequency

Modulating Frequency

Ring Modulation

- Formally, we are performing the following mathematical function. This is also called ring modulation:

$$R(t) = C(t) * M(t)$$

- Above, C stands for the carrier function based on time and M stands for our modulating function based on time. Let's assume we are performing ring modulation on two sine waves. Then the following formulas would satisfy that condition.

$$R(t) = A_c sin(2\pi f_c t + \ \phi_c) * A_m sin(2\pi f_m t + \ \phi_m)$$

- Here, A_c stands for the amplitude of the carrier wave. ϕ_c refers to phase of the carrier wave. w_c refers to the angular frequency. Ditto for the modulating frequency variables. Time is measured in seconds.

Checking Our Math

```
{SinOsc.ar(440, 0, 0.5)*SinOsc.ar(220, pi, 1)}.play;
```

$$R(t) = A_c sin(2\pi f_c t + \phi_c) * A_m sin(2\pi f_m t + \phi_m)$$

$A_c = 0.5$
$f_c = 440$
$\phi_c = 0$
$A_m = 1$
$f_m = 220$
$\phi_m = \pi$

$$R(t) = 0.5sin(880\pi t) * sin(440\pi t + \pi)$$

These are the samples that are computed on the server.

Understanding Ring Modulation

- Let's assume our sine waves are in phase for simplicity sake. How can we explain the issue of detuning?
- We will need to use one trigonometric identity to help separate the multiplication. Ideally, we would like to see our signal as a sum of sine waves.

$$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

Expanding Ring Modulation Formula

For simplicity's sake, let's consider two sine waves in phase.

$$A_c \sin(2\pi f_c t) * A_m \sin(2\pi f_m t)$$

$$A_c A_m[\sin(2\pi f_c t)\sin(2\pi f_m t)]$$

$$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

$$A_c A_m[\frac{\cos(2\pi f_m t - 2\pi f_c t) - \cos(2\pi f_m t + 2\pi f_c t)}{2}]$$

$$\frac{A_c A_m}{2}[\sin\left(2\pi(f_m - f_c)t + \frac{\pi}{2}\right) + \sin\left(2\pi(f_m + f_c)t - \frac{\pi}{2}\right)]$$

Note that because the signals are originally multiplied we can write the equation like this as well

$$\frac{A_c A_m}{2}[\sin\left(2\pi(f_c - f_m)t + \frac{\pi}{2}\right) + \sin\left(2\pi(f_c + f_m)t - \frac{\pi}{2}\right)]$$

Takeaways

$$\frac{A_c A_m}{2}\left[\sin\left(2\pi(f_c - f_m)t + \frac{\pi}{2}\right) + \sin\left(2\pi(f_c + f_m)t\ - \frac{\pi}{2}\right)\right]$$

- Ring Modulation produces two sine waves whose frequencies are centered around the carrier frequency but do **not** include any of the original frequencies.
- Revisiting the tremolo effect with a LFO for the modulating frequency, we can see that the resulting frequencies are slightly above and below the carrier frequency. As the split widens we hear the detuning and when the split exceeds about 15 - 20Hz, we start to hear the frequencies as separate sounds.
- Note that the maximum amplitude is cut in half and is determined by the strength of both the carrier and the modulator.
- These two frequencies are referred to as sidebands.

https://www.desmos.com/calculator/wvy0ue6wsq

Sidebands

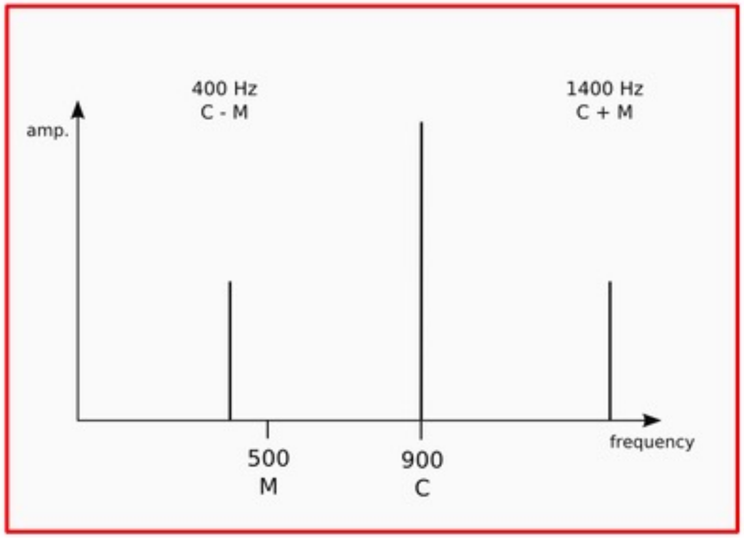


Chart taken from Sound Synthesis Theory

Practical Uses of Ring Modulation

- Ring modulation was first conceived in 1934 at Bell Labs for carrying voices over telephone cables.
- Introduction into music in the 1940s and 1950s. Used by Karlheinz Stockhausen, a pioneer of electronic music and experimental composer, used ring modulation in some of his works in *Gesang der Jünglinge.*
- Also popularized in jazz-fusion and classic rock songs from the 1970s
- Ring modulation has a kind of “metallic” sound and has also been used in early sci-fi scores.

Amplitude Modulation

- One downside to ring modulation is that we fail to preserve any of the original frequencies, either the carrier or the modulator, once the signals have been multiplied. How can we solve this issue?

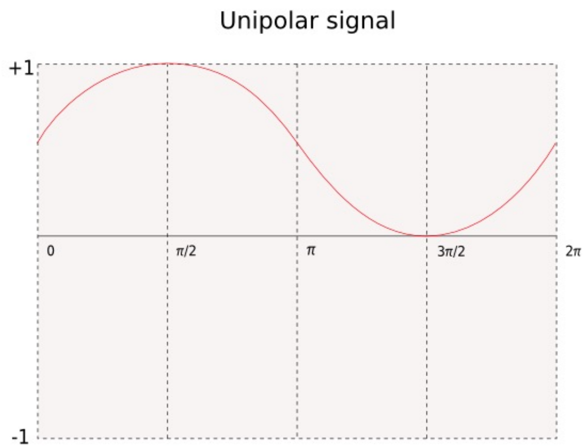
$$S(t) = C(t) * M(t) + C(t)$$

or

$$S(t) = C(t) * (M(t) + 1)$$

Unipolar/Bipolar

- Unipolar signals have amplitudes in the range from 0 to 1
- Bipolar signals have amplitudes in the range of -1 to 1
- In amplitude modulation, the modulator signal needs to be a unipolar signal to include the DC offset.



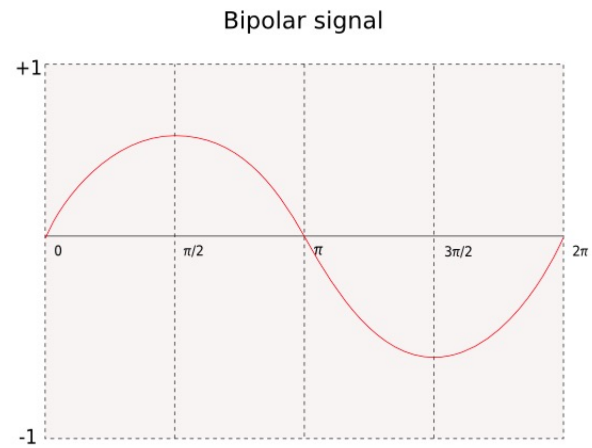


Chart taken from Sound Synthesis Theory

Amplitude Modulation

- What would the formula for amplitude modulation of two sine waves in phase look like? Our original formula for ring modulation plus the addition of the carrier frequency.

$$\frac{A_c A_m}{2}\left[\sin\left(2\pi(f_c - f_m)t + \frac{\pi}{2}\right) + \sin\left(2\pi(f_c + f_m)t - \frac{\pi}{2}\right)\right] + A_c \sin(2\pi f_c t)$$

- One thing to consider is that because we have a DC offset and several partials added together, our signal will likely exceed the boundaries of -1 to +1 for amplitude. Quick solution is to halve the amplitude of our signal.

$$\frac{A_c A_m}{4}\left[\sin\left(2\pi(f_c - f_m)t + \frac{\pi}{2}\right) + \sin\left(2\pi(f_c + f_m)t - \frac{\pi}{2}\right)\right] + \frac{A_c}{2}\sin(2\pi f_c t)$$

https://www.desmos.com/calculator/7ivjtwxcf8

Amplitude Modulation

- Amplitude Modulation produces three tones: the carrier frequency and two sidebands centered around the carrier frequency but with half the amplitude.
- Both amplitude modulation and ring modulation are capable of producing aliasing if the sum of their frequencies exceed the Nyquist frequency.
- Both ring modulation and amplitude modulation need not use simple sine waves as carriers and modulators. Any waveforms could be used.

Modulation Index

- We would like the ability to control the amplitude of the sidebands. When both the carrier wave and the modulating wave have equal amplitudes, the sidebands will be half the amplitude of the carrier waves. When the amplitude of the modulating wave decreases, the amplitude of the sidebands decreases. We can express this relationship as a ratio of the peak amplitude of the modulating wave to the peak amplitude of carrier wave. We call this the modulation index.

$$m = \frac{A_m}{A_c}$$

- We always assume that $A_c \geq A_m$. Therefore, the modulation index ranges between 0 and 1. We will modify our original formula as such:

$$S(t) = C(t) * (\frac{M(t)}{A_c} + 1)$$

- Using two sine waves for both signals as we have already done, results in the following expression (now including the modulation index):

$$S(t) = 0.5 * A_c \sin(2\pi f_c t) * (A_c m \sin(2\pi f_m t) + 1)$$

Scale total amplitude from -1 to + 1

Same as A_m but now expressed as index

Aside: AM Radio

- Amplitude modulation is the main technique for AM Radio transmission.
- If you look at an AM radio, you'll see that you can tune your radio to specific frequencies, ranging from 540,000– 1,600,000Hz (i.e., 540kHz – 1,600kHz). Each station is assigned in increments of 10kHz.
- AM radio frequencies correspond to the carrier frequency of amplitude modulated signal.
- The modulator signal is the talk show, song, sports broadcast signal that we wish to transmit/broadcast and is embedded into the carrier signal through amplitude modulation.

Aside: AM Radio

- Why can't a song be directly sent as a signal? Why must it be modulated with radio waves (range from 3kHz – 300GHz)?
  - What happens if we all try and send signals in the range 20-20000Hz?
  - Can use smaller antennas with higher frequencies.
- Solution: encode signal in a higher frequency and broadcast it.
- Antennas, which have specific frequency ranges in part based on their length, pick up signals and sends them to a radio receiver which can parse out the desired frequency.
- The signal then needs to go through a process of "demodulation" to remove the carrier signal and extract the modulator signal (i.e., the signal we transmitted).

Aside: AM Radio

- There are several techniques for "demodulating" an AM signal.
- One trick is to multiply the signal by the same carrier frequency at the same phase (called a **product detector**).

$$S(t) = \sin(2\pi f t) * (M(t) + 1)$$

$$\begin{aligned} \textit{Modulating signal} &= \sin(2\pi f t) * \sin(2\pi f t) * (M(t) + 1) \\ &= \tfrac{1}{2} * (\cos(2\pi f t - 2\pi f t) - \cos(4\pi f t)) * (M(t) + 1) \\ &= \tfrac{1}{2} * (1 - \cos(4\pi f t)) * (M(t) + 1) \\ &= \frac{1}{2}(M(t) + 1) - \frac{1}{2}\cos(4\pi f t)(M(t) + 1) \end{aligned}$$

DC offset can be subtracted out

Can be excised using filters. More soon.