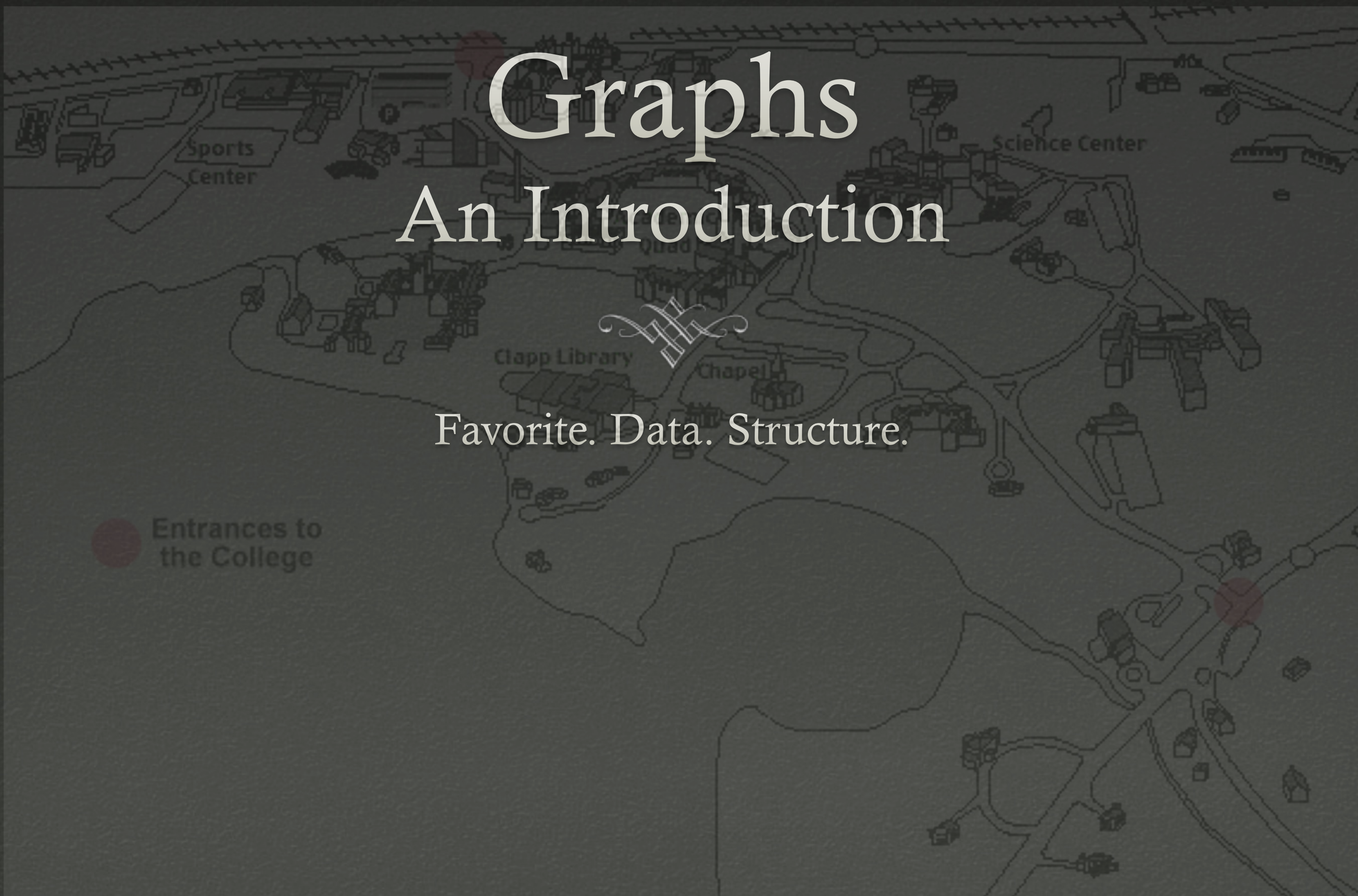
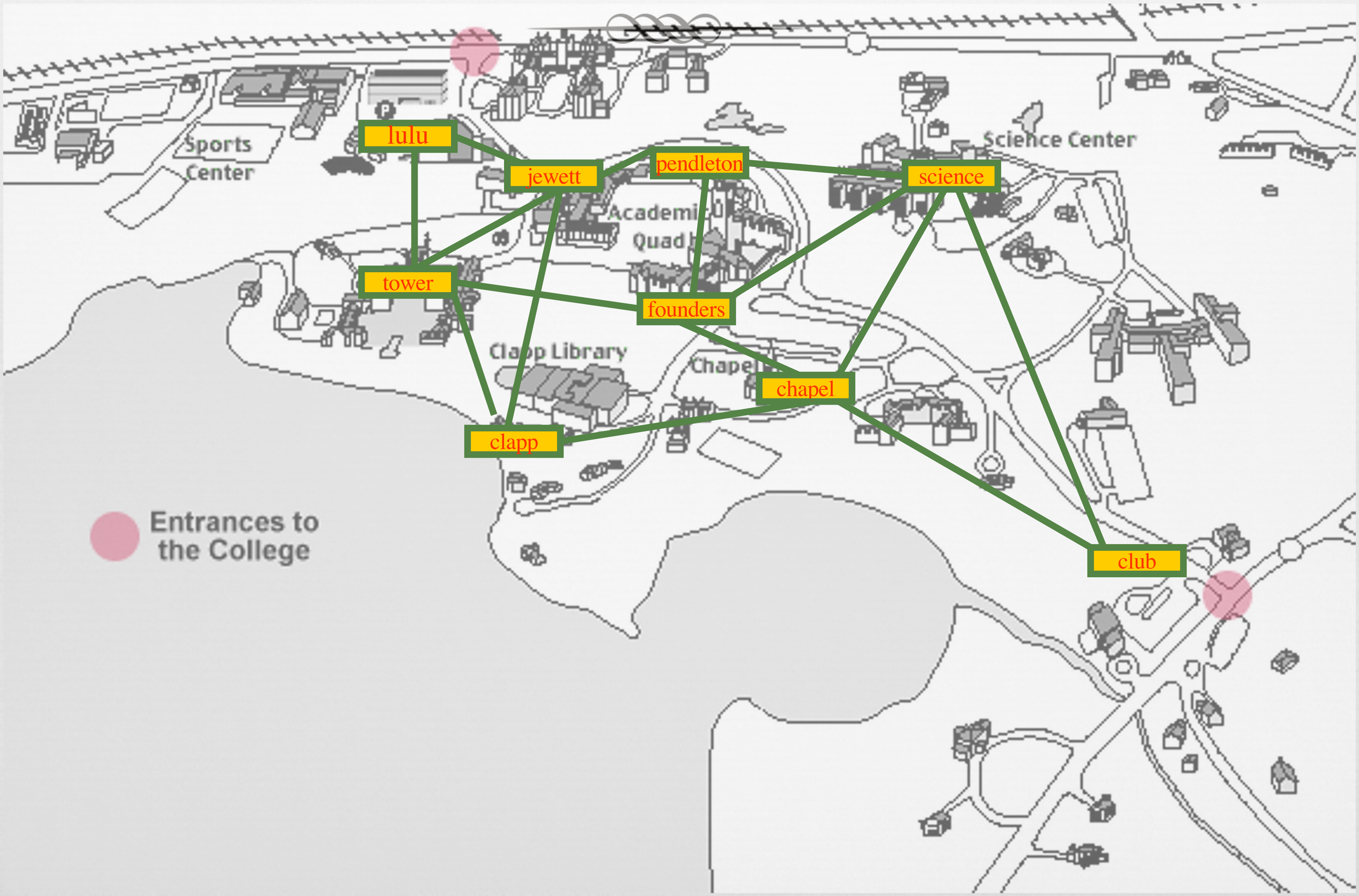
Graphs

An Introduction

Favorite. Data. Structure.

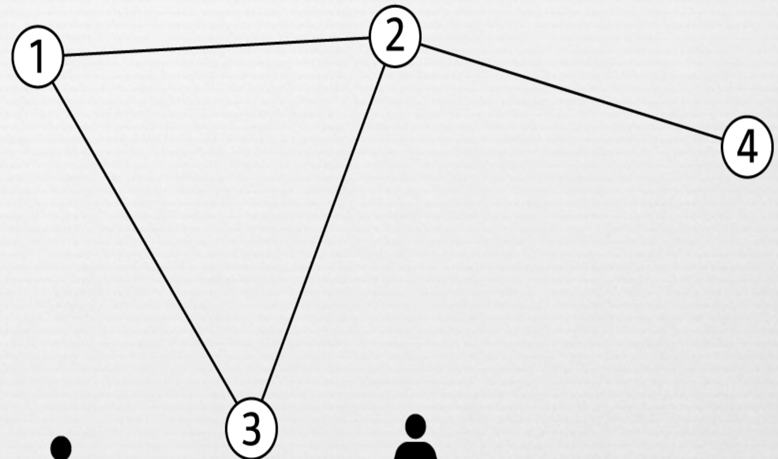


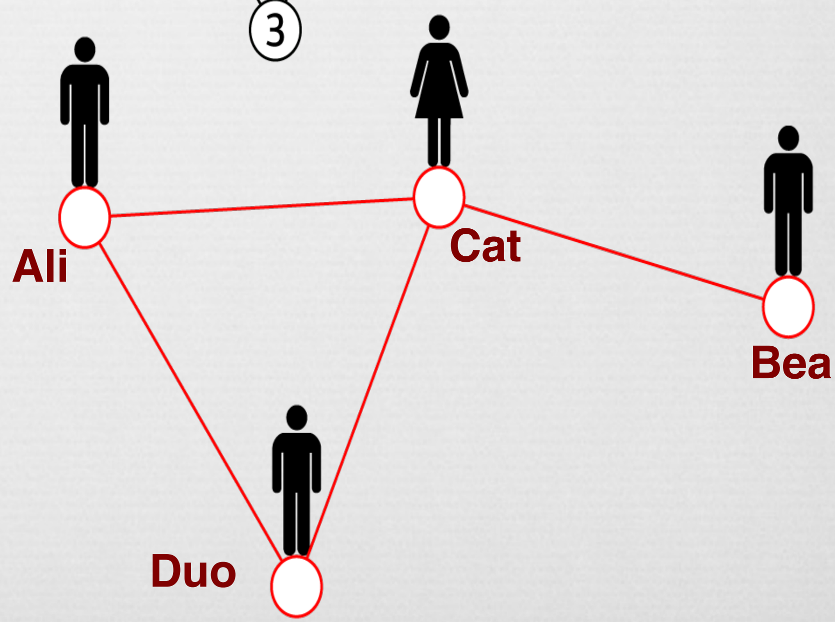
A familiar place…



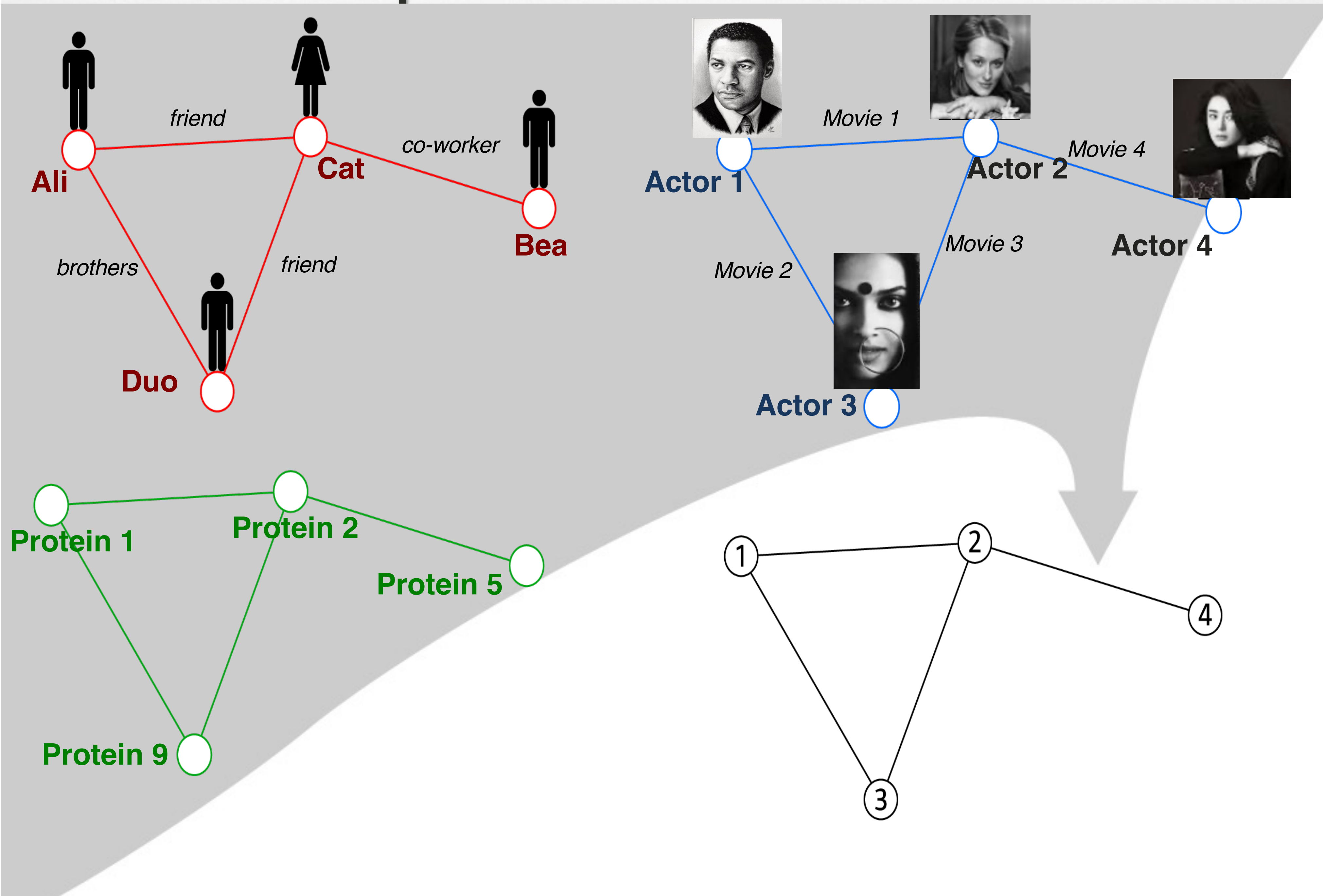
Graphs (and Networks)

- *Graphs* are made up of
  - ***nodes*** (or ***vertices***) and
  - ***connections*** between them (or ***edges***)
- Vertices typically have a name or label, e.g., 3 or Duo
- Edges are referenced by the pair of vertices they connect, e.g., (3,2) or (Duo, Cat)
- When nodes represent entities, connections represent relationships.
  - In this case graphs are also called *Networks.*



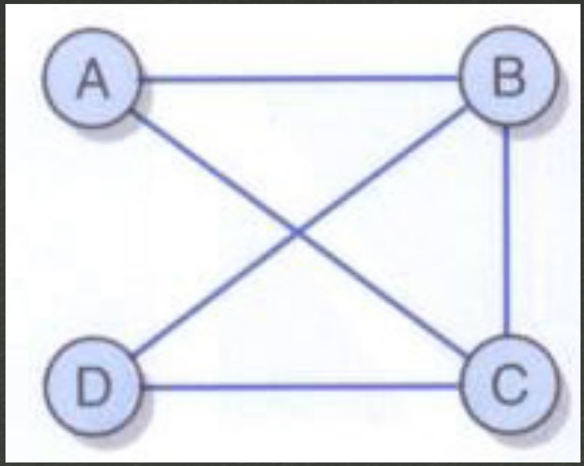


Graphs and Networks



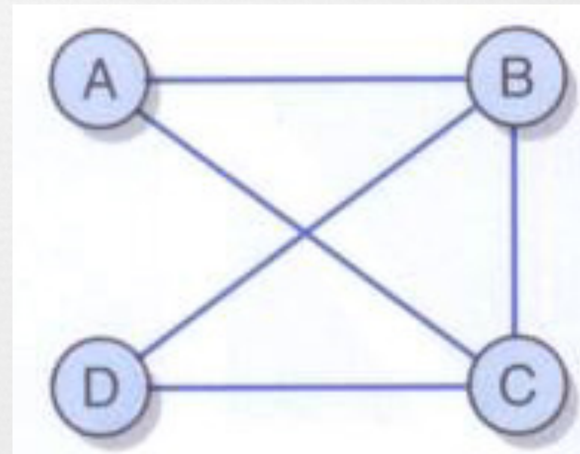
Undirected Graphs

Edges are bidirectional. Like two-way streets



(Undirected) Graph Definition

- Our first non-linear data structure!
- An **undirected graph** G consists of two sets G = {V, E}
  - A set of V **vertices**, or nodes
  - A set of E **edges**, relationships between nodes
- A **subgraph** G' consists of a subset of the vertices and edges of G
- **Adjacent** are two vertices connected by an edge
- An edge that connects a vertex to itself is called a *self-loop* or *sling.* We will avoid them.



V = {   }

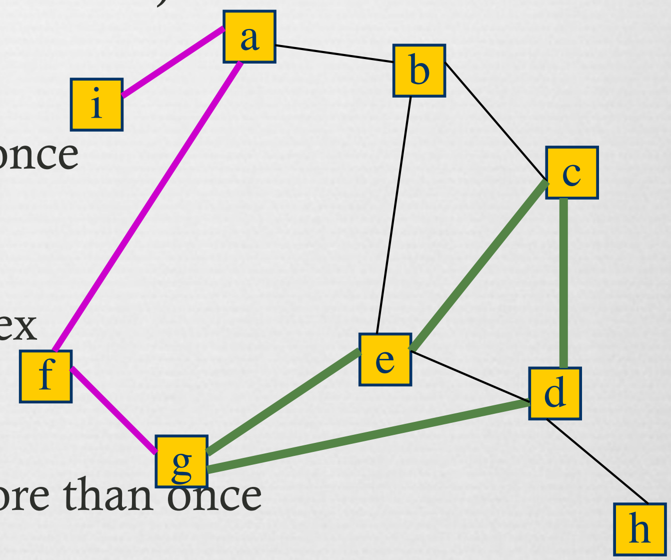
E = { (,), (,),
(,), (,), (,) }

Paths and Cycles

- A **path** between two vertices is a sequence of edges that begins at the first vertex and ends at the other vertex

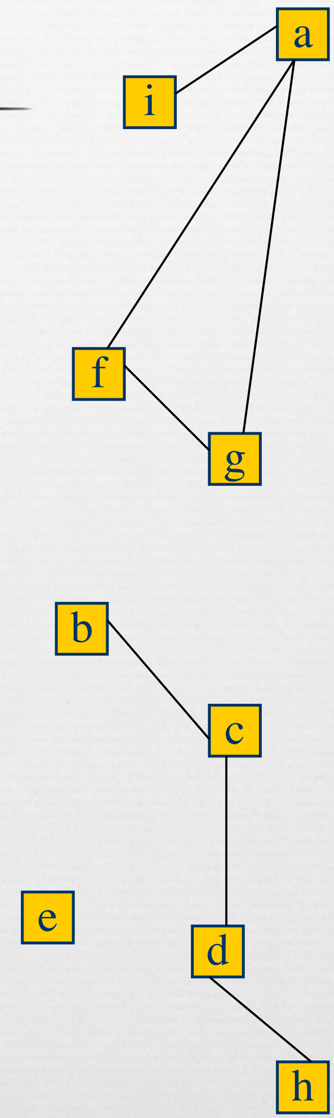
(The edges in the path could be required to be distinct or not.)

- A **simple path**
  - is a path that passes through a vertex at most once
- A **cycle**
  - is a path that begins and ends at the same vertex
- A **simple cycle**
  - A cycle that does not pass through a vertex more than once
- A graph that has no cycle is called **Acyclic**

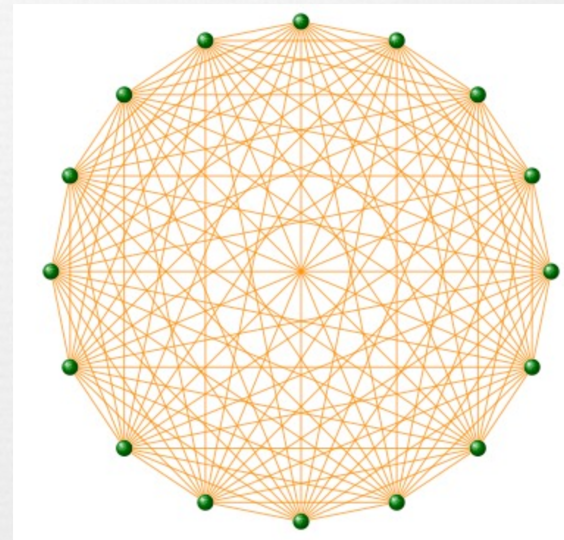


(undirected) Graph Connectivity

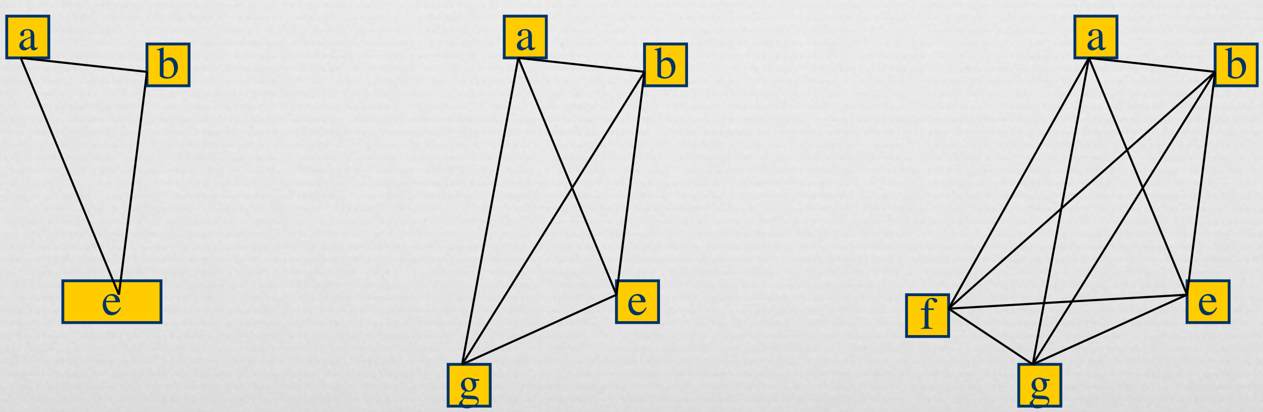
- A **connected** graph
  is a graph that has a **path** between each pair of vertices
- A **disconnected** graph
  is a graph that has at least one pair of vertices without a path between them
- A **connected component** is a connected subgraph of the graph



Complete Graph



- A **complete** graph
  - is a graph that has an **edge** between every pair of distinct vertices
- How many edges does a complete graph with **n** vertices have?

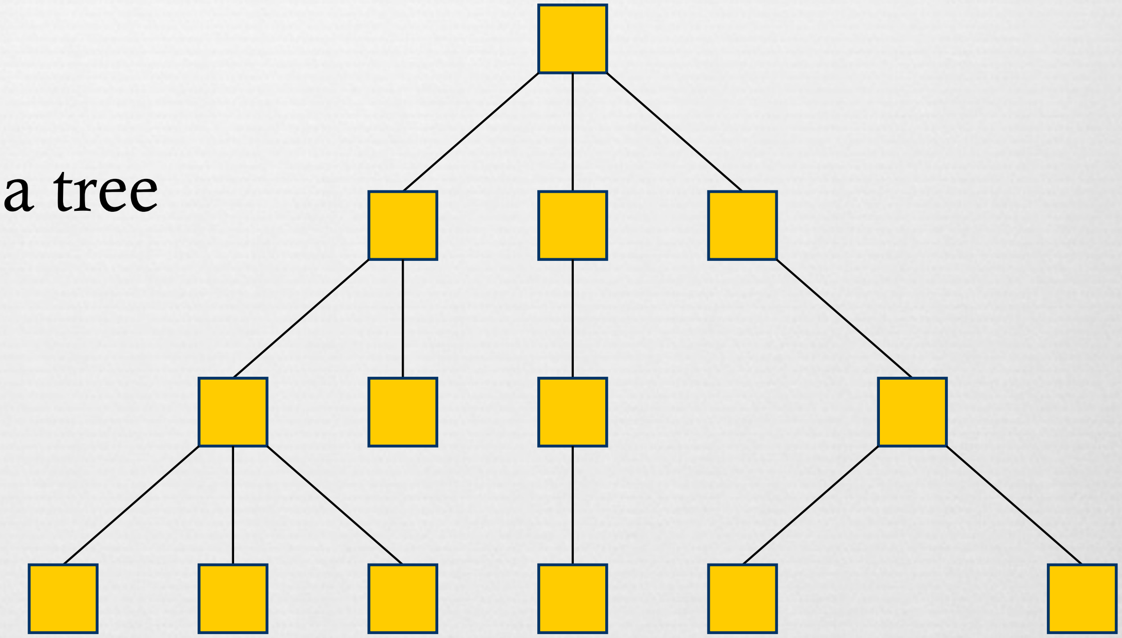


Tree

a connected Graph without cycles

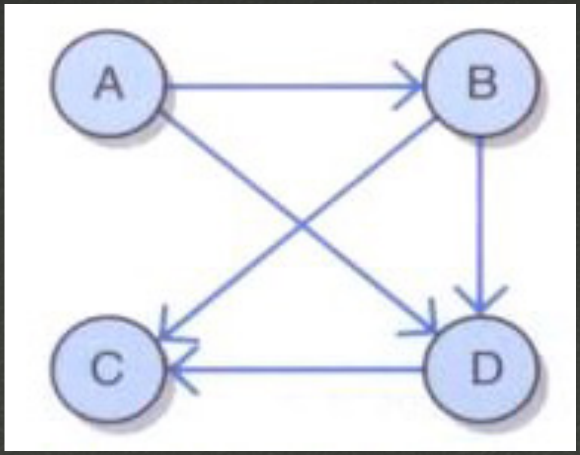
How many simple paths are there between two ***tree*** nodes?

How many **edges** does a tree with n nodes have?

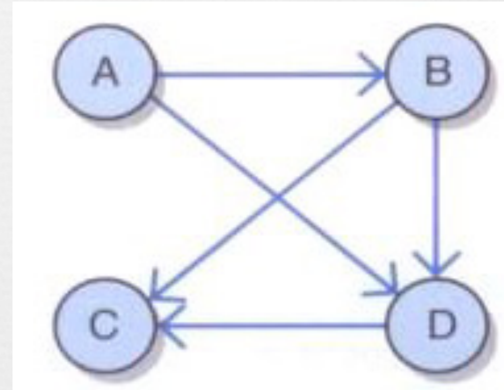


Directed Graphs
(aka: DiGraphs)

Edges ("arcs") are uni-directional. Like one-way streets



Directed Graphs and DAGs



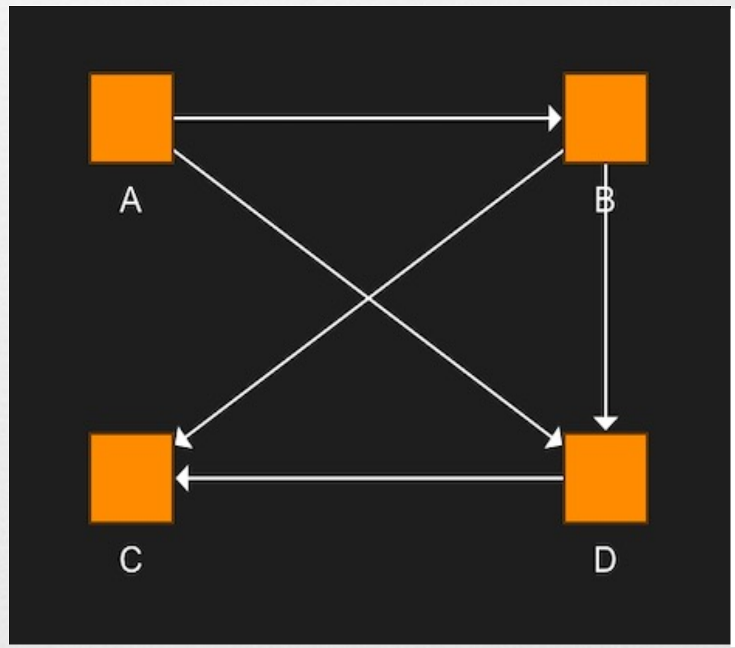
- **Directed** graph G = {V, A}
  - **Arcs** (or **links**) are directed edges between vertices
  - A vertex y is **adjacent** to vertex x **iff** (if and only if) there is an arc (directed edge) from x to y
- Directed **path** is a sequence of arcs between two vertices
- Directed **cycle** is a directed path from a vertex to itself
- **Directed Acyclic Graph (DAG)** is a digraph without directed cycles
- You could turn
  a digraph into a DAG
  by removing some arcs to break cycles
  - How few arcs can you remove to turn this digraph into a DAG?

V = { }

A = { (,), (,),
(,), (,), (,) }

Visualizing Graphs with yEd

- yEd: A simple graph visualization
- Download it:
  `https://www.yworks.com/products/yed`
- You can create any graph by clicking (for vertices) and clicking-and-dragging (for edges)
- Lots of graph formats supported. Use `.tgf` for simplicity
- Once you upload a file, choose
  Layout > Circular
  to see it laid out nicely.
  Explore more layouts for fun!

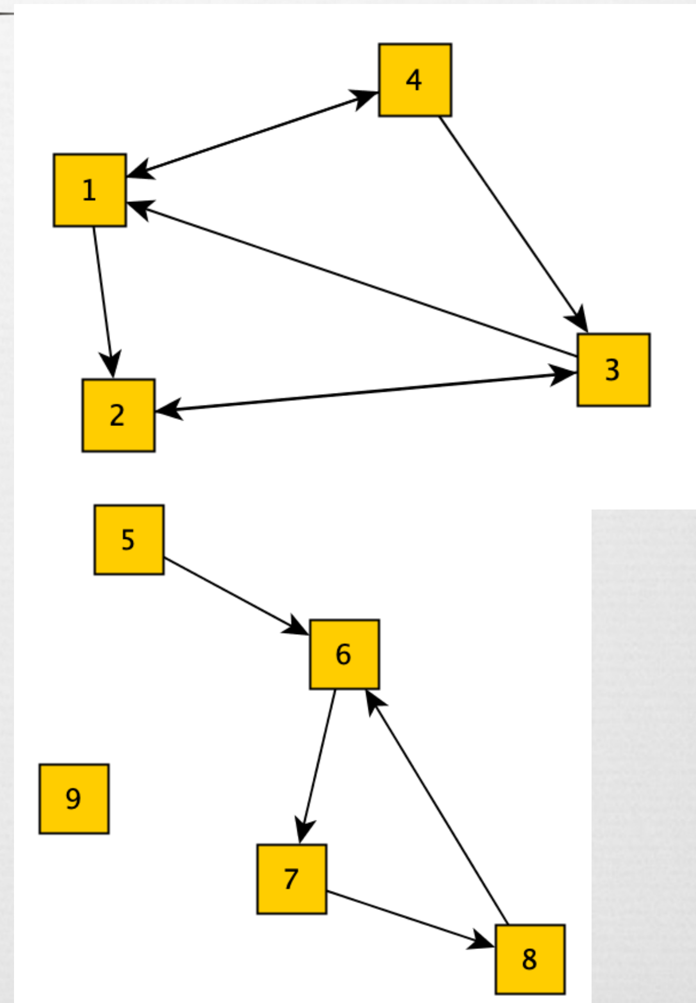


tgf

```
1 A
2 B
3 D
4 C
#
1 2
2 3
3 4
2 4
1 3
```

DiGraph
Strong Connectivity

- A **strongly connected** graph
  - A graph that has a
    directed path
    between any pair of vertices
- A **strongly connected component** of a graph
  - a *maximally*
    strongly connected subgraph
- How many strongly connected components do you see in this digraph?



Implementing Graphs

An **undirected** graph G consists of two sets G = {V, E},
a set V of vertices and
a set E of edges.

A **di**graph G consists of two sets G = {V, A},
a set V of vertices and
a set A of arcs (directed edges)

```
public interface DiGraph<T> {

public int getNumVertices()   //  Returns number of vertices
public int getNumArcs()       //  Returns the number of arcs

public void addVertex(T v)    // Insert a vertex in a graph
public void removeVertex(T v) // Delete a vertex along with
                       any arcs between v and other vertices

public void addArc(T v1, T v2)   // Adds an arc from v1->v2

public void removeArc(T v1, T v2) // Deletes the arc between
                                two given vertices in a graph

public boolean isArc(T v1, T v2)  // Returns true iff an arc
                           exists between vertices v1 and v2

public boolean isEmpty() // Returns true iff a graph is empty

public String toString() // Returns a String representation

public void saveToTGF(String fName) // Saves graph fName.tgf
```

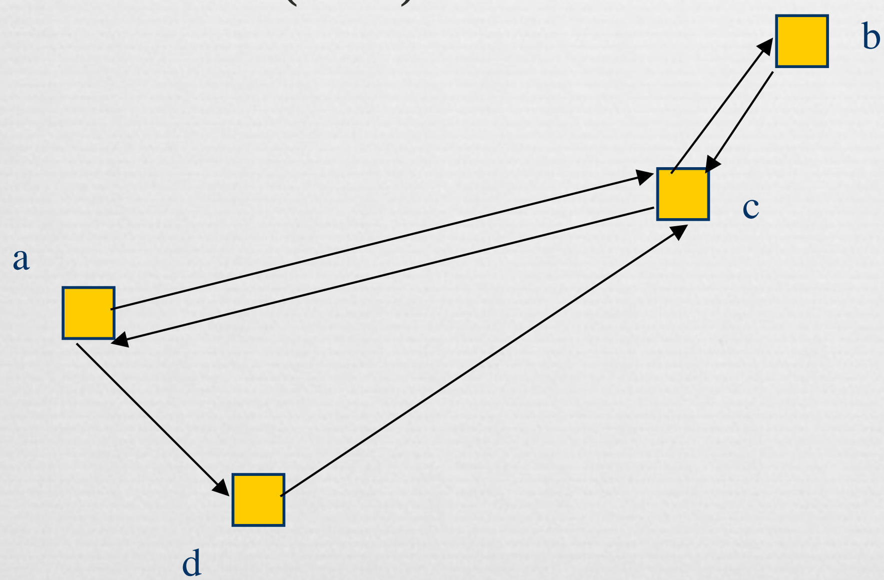
Implementing (Di)Graphs with Adjacency Matrix

NOTE: If a **digraph** has
between every pair of vertices
either *both* arcs or *none*,
then it can be considered **undirected**

Arcs	0	1	2	3
0	0	0	1	1
1	0	0	1	0
2	1	1	0	0
3	0	0	1	0

Adjacency Matrix

- **Adjacency matrix for digraph with**
  - ***n** vertices*: **numbered 0, 1, …, $n - 1$**
  - *arcs:* **boolean $n \times n$ array where *arcs[i][j]* =**
    - 1 (true) if there is an arc from vertex *i* to vertex *j*
    - 0 (false) if there is no arc from vertex *i* to vertex *j*



Vertices

0	1	2	3
a	b	c	d

Arcs

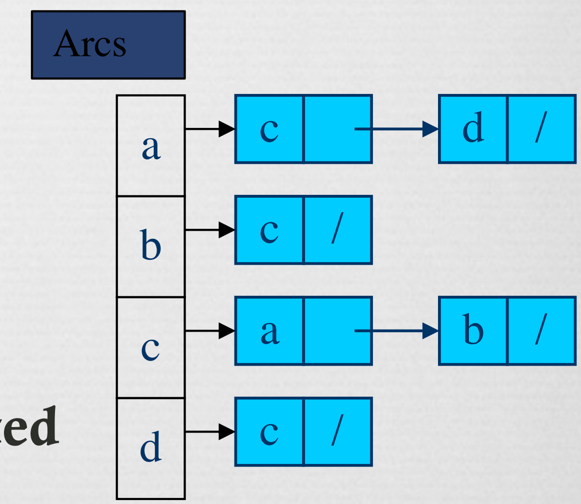
	0	1	2	3
0	0	0	1	1
1	0	0	1	0
2	1	1	0	0
3	0	0	1	0

What do you need to add to turn this digraph into an undirected graph?
What property does the matrix of an undirected graph have?

AdjMatDiGraph<T>

```
public class AdjMatDiGraph<T> implements DiGraph<T> {
  private final int DEFAULT_CAPACITY = 10;

private boolean[][] arcs;    // adjacency matrix of arcs
  private T[] vertices;       // array of vertices (could be a Vector)
  private int n;              // number of vertices in the graph
public AdjMatGraph(){ // constructor
   this.n = 0;
   this.arcs = new boolean[DEFAULT_CAPACITY][DEFAULT_CAPACITY];
   this.vertices = (T[])(new Object[DEFAULT_CAPACITY]);
}
public boolean isEmpty(){… // returns true if a graph is empty

}
public int getNumVertices(){… // returns the number of vertices
}
public int getNumArcs(){… //returns the number of arcs
//count them!
}                                                           etc…
```

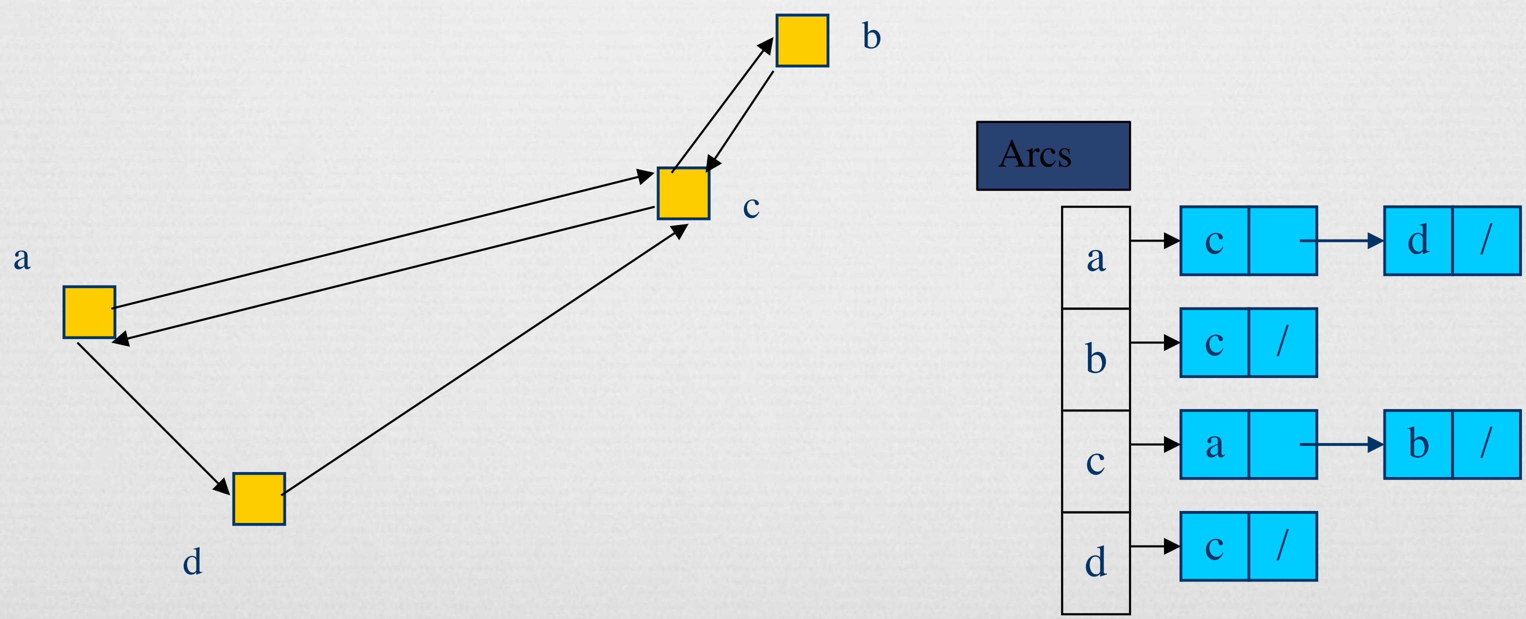
Implementing (Di)Graphs with Adjacency Lists



NOTE: If a **digraph** has between every pair of vertices either *both* arcs or *none*, then it can be considered **undirected**

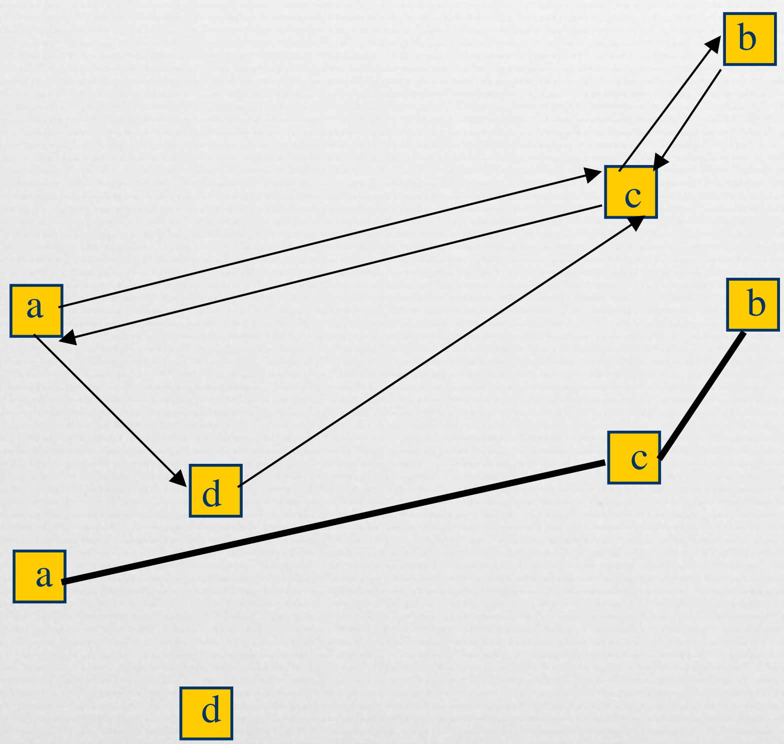
Adjacency Lists

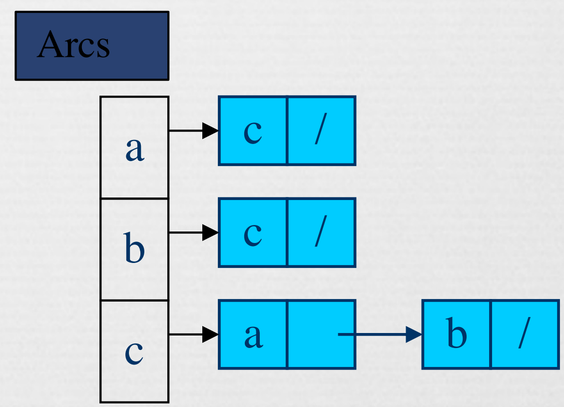
- **An adjacency list for a DiGraph with**
  - ***n*** *vertices* **numbered 0, 1, …, *n* – 1**
  - *arcs*: array (or Vector) of *n* linked lists
  – The *i*th linked list has a list entry for vertex *j* iff the graph contains an arc from vertex *i* to vertex *j*



Undirected & Directed Graph Representation

- We can use either **`AdjMatDiGraph`** or **`AdjListDiGraph`** to represent both undirected and directed graphs.
- In an undirected graph every edge v–w appears as two arcs v ->w and w->v in the adjacency lists





- What do you need to add to turn this digraph into an undirected graph?

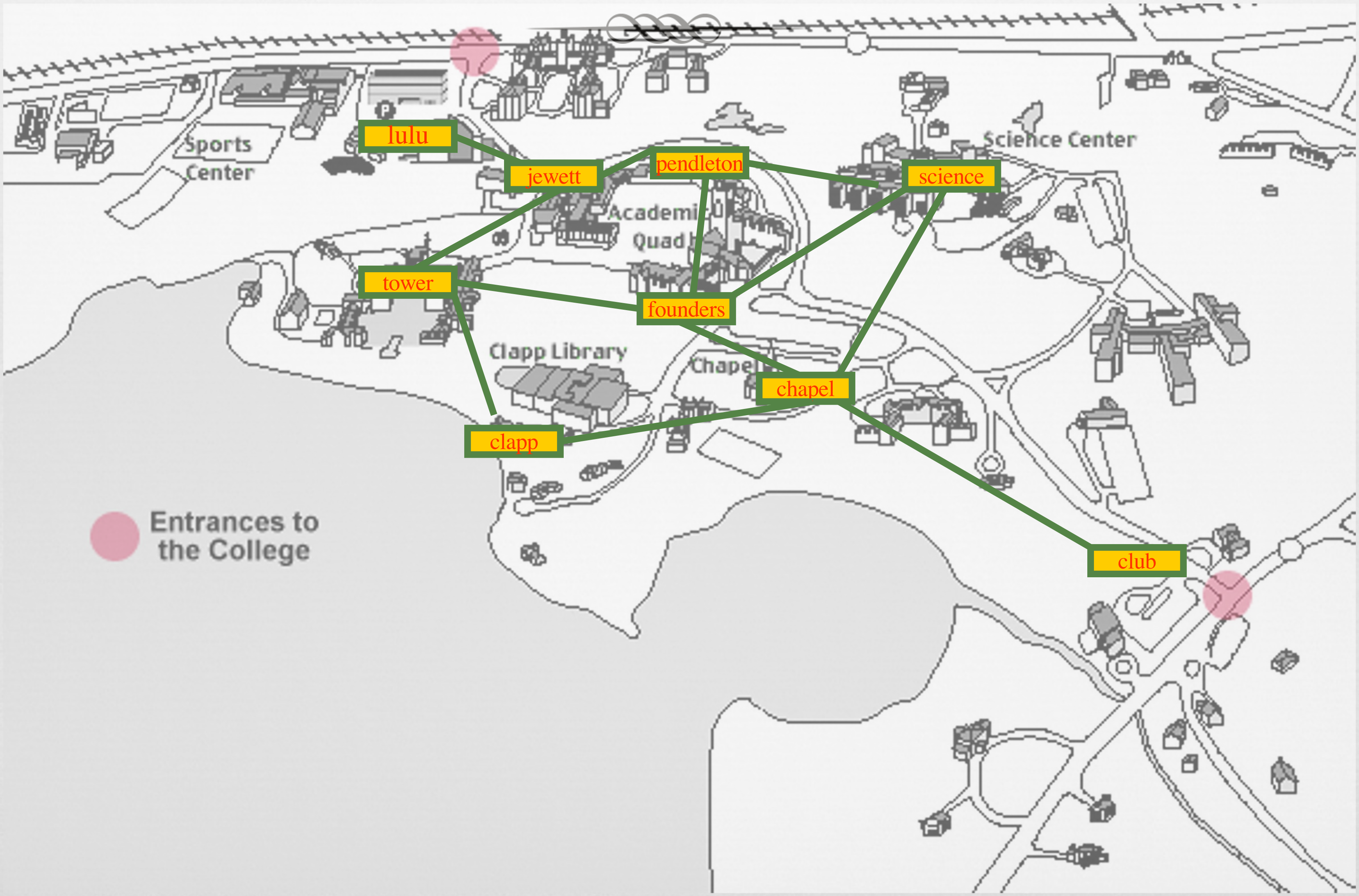
AdjListDiGraph<T>

```
public class AdjListDiGraph<T> implements DiGraph<T> {
  private Vector<T> vertices;
  private Vector<LinkedList<T>> arcs;    // adjacency lists of arcs

public AdjListDiGraph(){ // constructor
  this.arcs = new Vector<LinkedList<T>>();
  this.vertices = new Vector<T>();
}
public boolean isEmpty(){… // returns true if a graph is empty

}
public int getNumVertices(){… // returns the number of vertices
}
public int getNumArcs(){… //returns the number of arcs
//count them!
}
```

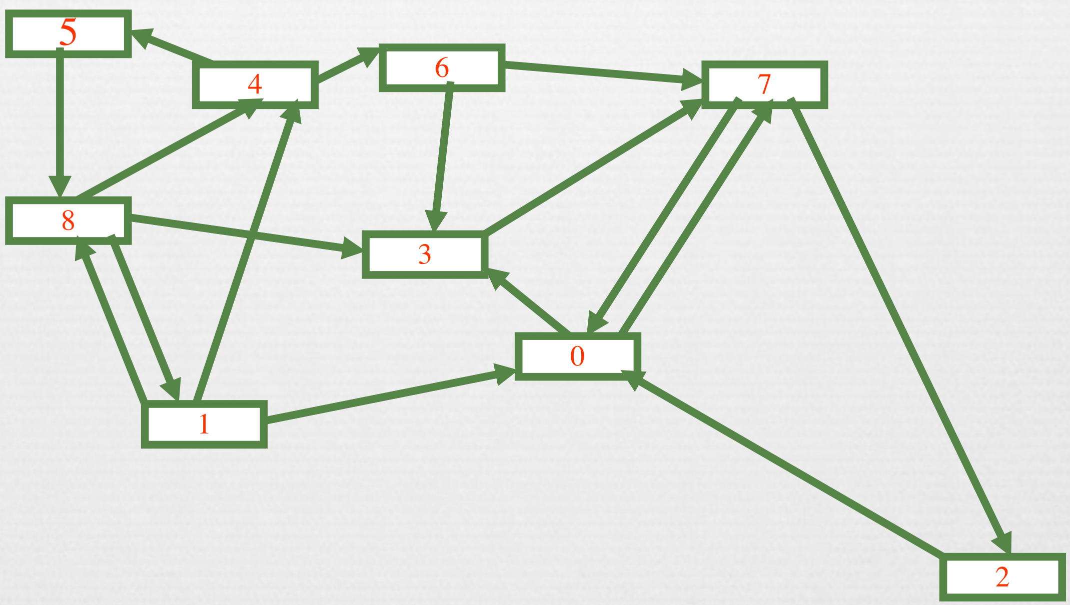
etc…

Practicing with the Wellesley Campus

WC Campus Undirected Graph



WC Campus DiGraph



WC Campus DAG