Graphs with negative weights

- Given a graph $G = (V,E)$ with a weight function $w: V \times V \to \mathbb{R}$
  - In other words, it could have negative weights

- Can we use Dijkstra’s algorithm to find shortest paths in a graph with negative edge weights?
Another example

• What about the shortest path between $s$ and $t$ here?

Negative cycles

Def. A negative cycle is a directed cycle such that the sum of its edge weights is negative.

$a$ negative cycle $W$: $c(W) = \sum_{e \in W} c_e < 0$
Shortest paths and negative cycles

If some path from \( v \) to \( t \) contains a negative cycle, then there does not exist a cheapest path from \( v \) to \( t \).

If \( G \) has no negative cycles, then there exists a cheapest path from \( v \) to \( t \) that is simple (and has \( \leq n - 1 \) edges).

How can we solve the Shortest Path problem as a dynamic program

Let's think together
**Shortest-Paths** $(V, E, c, t)$

**Foreach** node $v \in V$

\[ M[0, v] \leftarrow \infty. \]

\[ M[0, t] \leftarrow 0. \]

**For** $i = 1$ to $n - 1$

**Foreach** node $v \in V$

\[ M[i, v] \leftarrow M[i-1, v]. \]

**Foreach** edge $(v, w) \in E$

\[ M[i, v] \leftarrow \min \{ M[i, v], M[i-1, w] + c_{vw} \}. \]

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**Bellman-Ford** $(V, E, c, t)$

**Foreach** node $v \in V$

\[ d(v) \leftarrow \infty. \]

\[ \text{successor}(v) \leftarrow \text{null}. \]

\[ d(t) \leftarrow 0. \]

**For** $i = 1$ to $n - 1$

**Foreach** node $w \in V$

If $(d(w)$ was updated in previous iteration)

**Foreach** edge $(v, w) \in E$

If $(d(v) > d(w) + c_{vw})$

\[ d(v) \leftarrow d(w) + c_{vw}. \]

\[ \text{successor}(v) \leftarrow w. \]

If no $d(w)$ value changed in iteration $i$, **STOP**.