

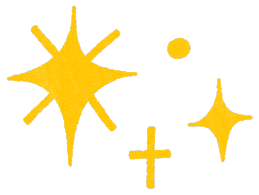
---

CS 232:  
Artificial Intelligence

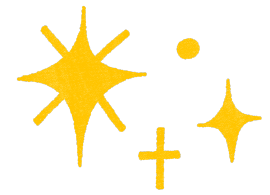
Fall 2023

---

Prof. Carolyn Anderson  
Wellesley College



# Reminders



- ◆ I have help hours Monday from 4-5:30pm
- ◆ Lepei's help hours: Sundays 6-8pm
- ◆ Lyra's help hours: Wednesdays 2-4pm
- ◆ Reading for next Tuesday: YLLATAILY 3-4



Recap

# Defining A Search Problem

**States:** a representation of physical configuration

**Nodes:** a data structure representing:

$\langle \text{state}, \text{parent-node}, \text{children}, \text{action}, \text{path-cost}, \text{depth} \rangle$

**Goal:** the state(s) we're trying to reach

**Start state:** initial starting point

**Solution:** a sequence of states that take us from the start state to the goal state.

**Optimal Solution:** the shortest solution

# Graph Search vs Tree Search

**function** TREE-SEARCH(*problem*) **returns** a solution, or failure

initialize the frontier using the initial state of *problem*

**loop do**

if the frontier is empty **then return** failure

choose a leaf node and remove it from the frontier

if the node contains a goal state **then return** the corresponding solution

expand the chosen node, adding the resulting nodes to the frontier

**function** GRAPH-SEARCH(*problem*) returns a solution, or failure

initialize the frontier using the initial state of *problem*

*initialize the explored set to be empty*

**loop do**

if the frontier is empty **then return** failure

choose a leaf node and remove it from the frontier

if the node contains a goal state **then return** the corresponding solution

*add node to the explored set*

expand the chosen node, adding the resulting nodes to the frontier

*only if not in the frontier ~~or~~ explored set*

OR

# Search Strategies

Review: *Strategy* = order of tree expansion

- Implemented by different queue structures (LIFO, FIFO, priority)

Dimensions for evaluation

- *Completeness* - always find the solution?
- *Optimality* - finds a least cost solution (lowest path cost) first?
- *Time complexity* - # of nodes generated (*worst case*)
- *Space complexity* - # of nodes simultaneously in memory (*worst case*)

Time/space complexity variables

- $b$ , *maximum branching factor* of search tree
- $d$ , *depth* of the shallowest goal node
- $m$ , maximum length of any path in the state space (potentially  $\infty$ )

# Uninformed Search

Uses only information available in problem definition

Informally:

***Uninformed search:*** All non-goal nodes in frontier look equally good

***Informed search:*** Some non-goal nodes can be ranked above others.

# Breadth-first search

Idea:

- Expand *shallowest* unexpanded node

Implementation:

- *frontier* is FIFO (First-In-First-Out) Queue:
  - Put successors at the *end* of *frontier* successor list.



# Properties of breadth-first search

**Complete?** yes (if  $f$  is finite)

**Optimal?** yes! (assuming we measure cost as # of steps)

**Time Complexity?**  $O(b^d)$

**Space Complexity?**  $O(b^d)$

# Exponential Space (and time) Is Not Good...

- Exponential complexity uninformed search problems *cannot* be solved for any but the smallest instances.
- (*Memory* requirements are a bigger problem than *execution* time.)

DEPTH	NODES	TIME	MEMORY
<i>2</i>	<i>110</i>	<i>0.11 milliseconds</i>	<i>107 kilobytes</i>
<i>4</i>	<i>11110</i>	<i>11 milliseconds</i>	<i>10.6 megabytes</i>
<i>6</i>	<i>10<sup>6</sup></i>	<i>1.1 seconds</i>	<i>1 gigabytes</i>
<i>8</i>	<i>10<sup>8</sup></i>	<i>2 minutes</i>	<i>103 gigabytes</i>
<i>10</i>	<i>10<sup>10</sup></i>	<i>3 hours</i>	<i>10 terabytes</i>
<i>12</i>	<i>10<sup>12</sup></i>	<i>13 days</i>	<i>1 petabytes</i>
<i>14</i>	<i>10<sup>14</sup></i>	<i>3.5 years</i>	<i>99 petabytles</i>

Assumes  $b=10$ , 1M nodes/sec, 1000 bytes/node

# Depth-First Search

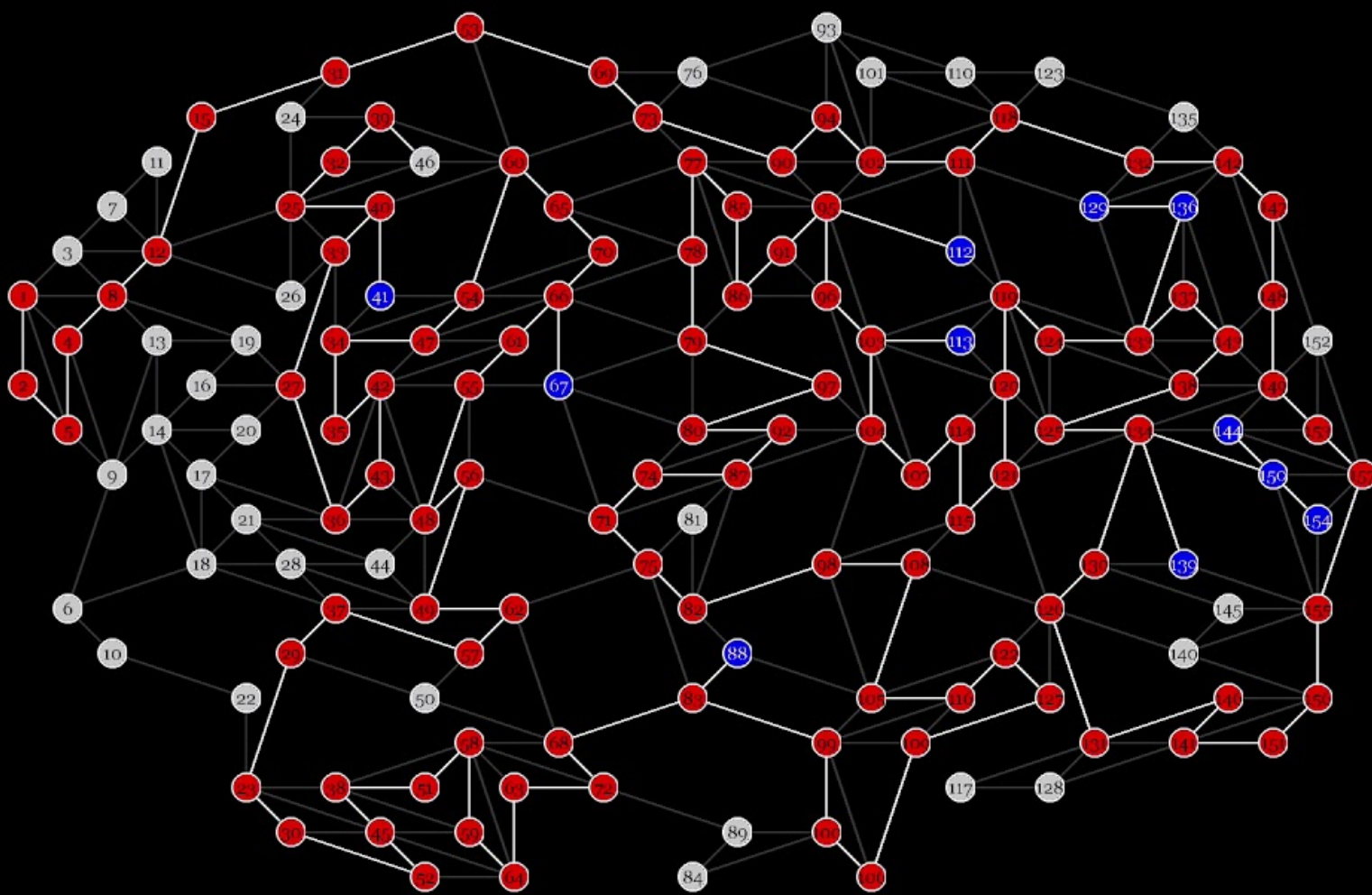
# Depth-first search

Idea:

- Expand *deepest* unexpanded node

Implementation:

- *frontier* is LIFO (Last-In-First-Out) Queue:
  - Put successors at the *front* of *frontier* successor list.



- current x
- discovered y
- node done
- Undiscovered edge
- - - Discovered edge

```

1 x = start vertex(1)
2 dfs(x)
3
4 def dfs(x):
5   mark x as visited
6   for each y in x connections:
7     if y not visited then
8       dfs(y)

```

Please subscribe @youtube.com/gjenkinslbcc or with icon in lower right >>>

# Properties of depth-first search

**Complete?** yes if tree is finite

**Optimal?** No

**Time Complexity?**

$$O(b^m)$$

$m =$  maximum  
depth of search  
space

**Space Complexity?**

$$O(b * m)$$

# Depth-first vs Breadth-first

Use depth-first if

- *Space is restricted*
- There are many possible solutions with long paths and wrong paths are usually terminated quickly
- Search can be fine-tuned quickly

Use breadth-first if

- *Possible infinite paths*
- Some solutions have short paths
- Can quickly discard unlikely paths

# Search Conundrum

## Breadth-first

- ✓ Complete,
- ✓ Optimal
- ✗ *but uses  $O(b^d)$  space*

## Depth-first

- ✗ Not complete *unless  $m$  is bounded*
- ✗ Not optimal
- ✗ Uses  $O(b^m)$  time; terrible if  $m \gg d$
- ✓ *but only uses  $O(b*m)$  space*



# Depth-limited search: A building block

Depth-First search *but with depth limit  $l$ .*

- i.e. nodes at depth  $l$  *have no successors.*
- No infinite-path problem!

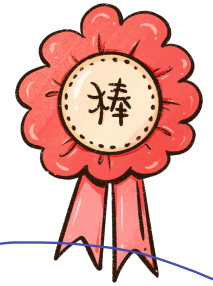
If  $l = d$  (by luck!), then optimal

- But:
  - If  $l < d$  then incomplete 😞
  - If  $l > d$  then not optimal 😞

Time complexity:  $O(b^l)$

Space complexity:  $O(bl)$  😊

# Summary of algorithms



Criterion	Breadth-First	Depth-First	Depth-limited	Iterative deepening
Complete?	<b>YES</b>	<b>NO</b>	<b>NO</b>	<b>YES</b>
Time	$b^d$	$b^m$	$b^l$	$b^d$
Space	$b^d$	$bm$	$bl$	$bd$
Optimal?	<b>YES</b>	<b>NO</b>	<b>NO</b>	<b>YES</b>