CS 232: Artificial Intelligence

Fall 2023

Prof. Carolyn Anderson
Wellesley College

Recap

Search

We've seen two kinds of search strategies so far:

- Uninformed search
  - Breadth-first search
  - Depth-first search
- Informed search
  - Uniform cost search
  - A* search

Annotate nodes w/ cost from Start → Node

Priority Queue

Optimistic Heuristic

$$F(\text{Node}) = \underbrace{G(\text{Node})}_{\text{Start} \rightarrow \text{Node}} + \underbrace{H(\text{Node})}_{\underline{\text{Node} \rightarrow \text{Goal}}}$$

A* search example

ordered by cost

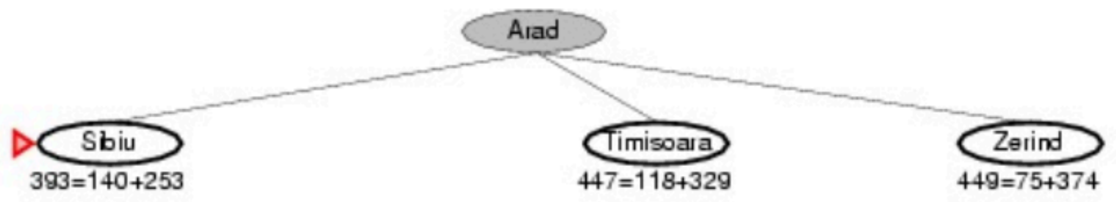
Frontier queue:

Sibiu 393

Timisoara 447

Zerind 449

Start



$f(N) = g(N) + h(N)$

$f(\text{Sibiu}) = g(\text{Sibiu}) + h(\text{Sibiu})$

cost/distance from Arad → Sibiu

guess of cost from Sibiu → Bucharest

distance between Sibiu → Bucharest

$= 140 + 253$

We add the three nodes we found to the Frontier queue.

We sort them according to the **g()+h()** calculation.

A* search example

ordered by cost

Frontier queue: Rimniu Vilcea 413

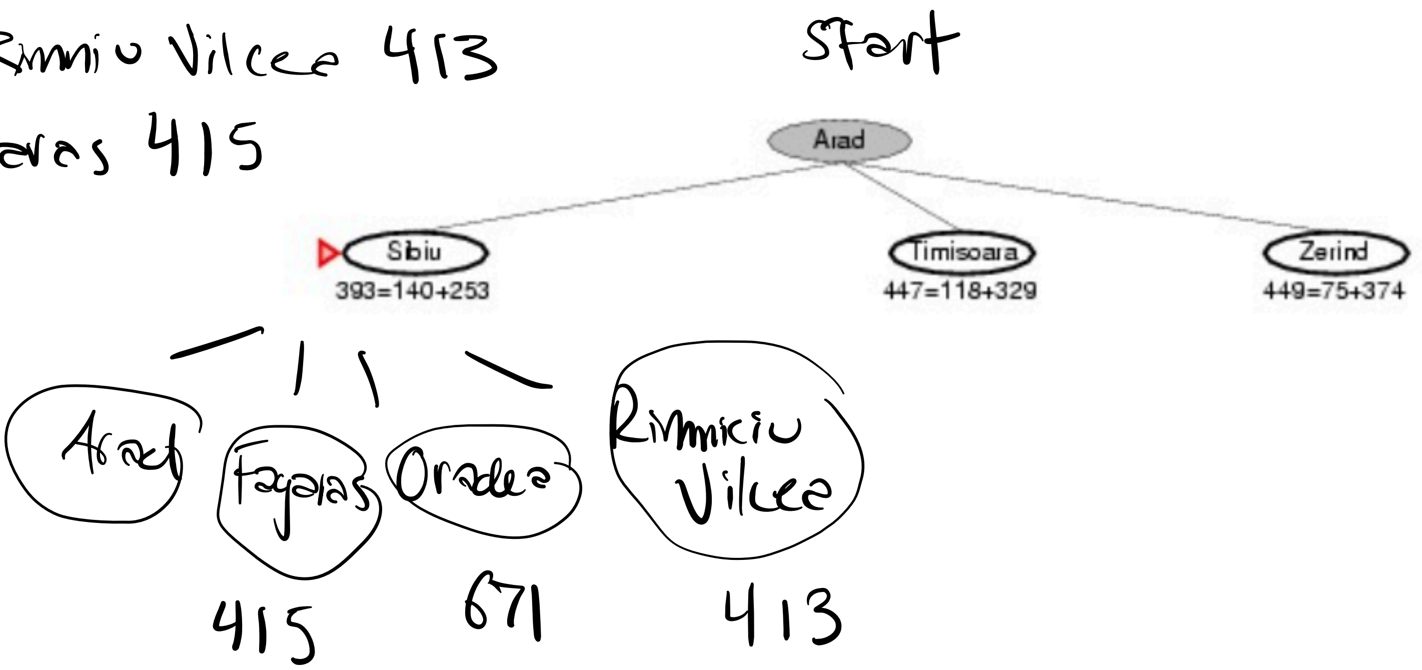
Sibiu 393 Fagaras 415

Timisoara 447

Zerind 449

Arad 646

Oradea 671



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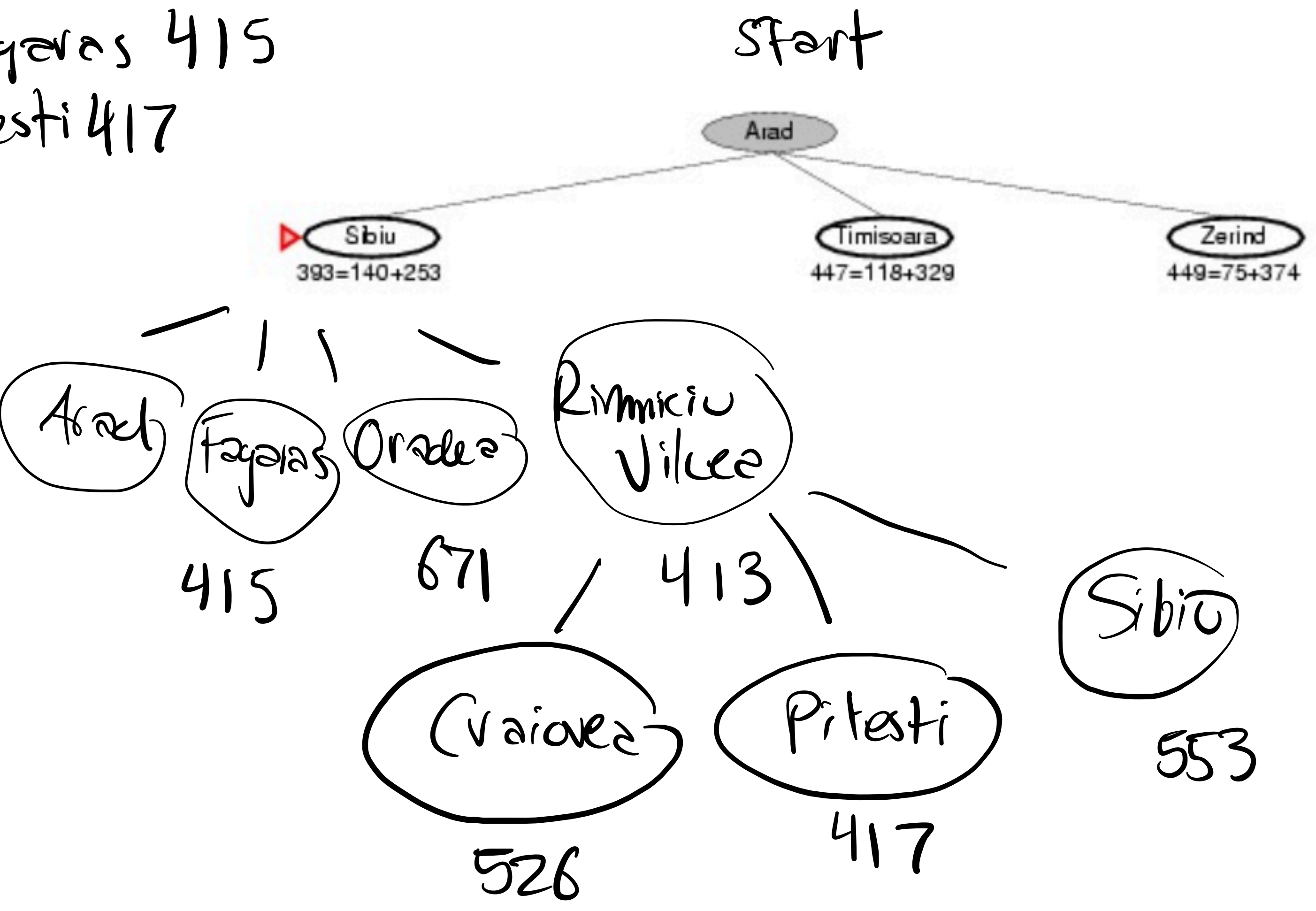

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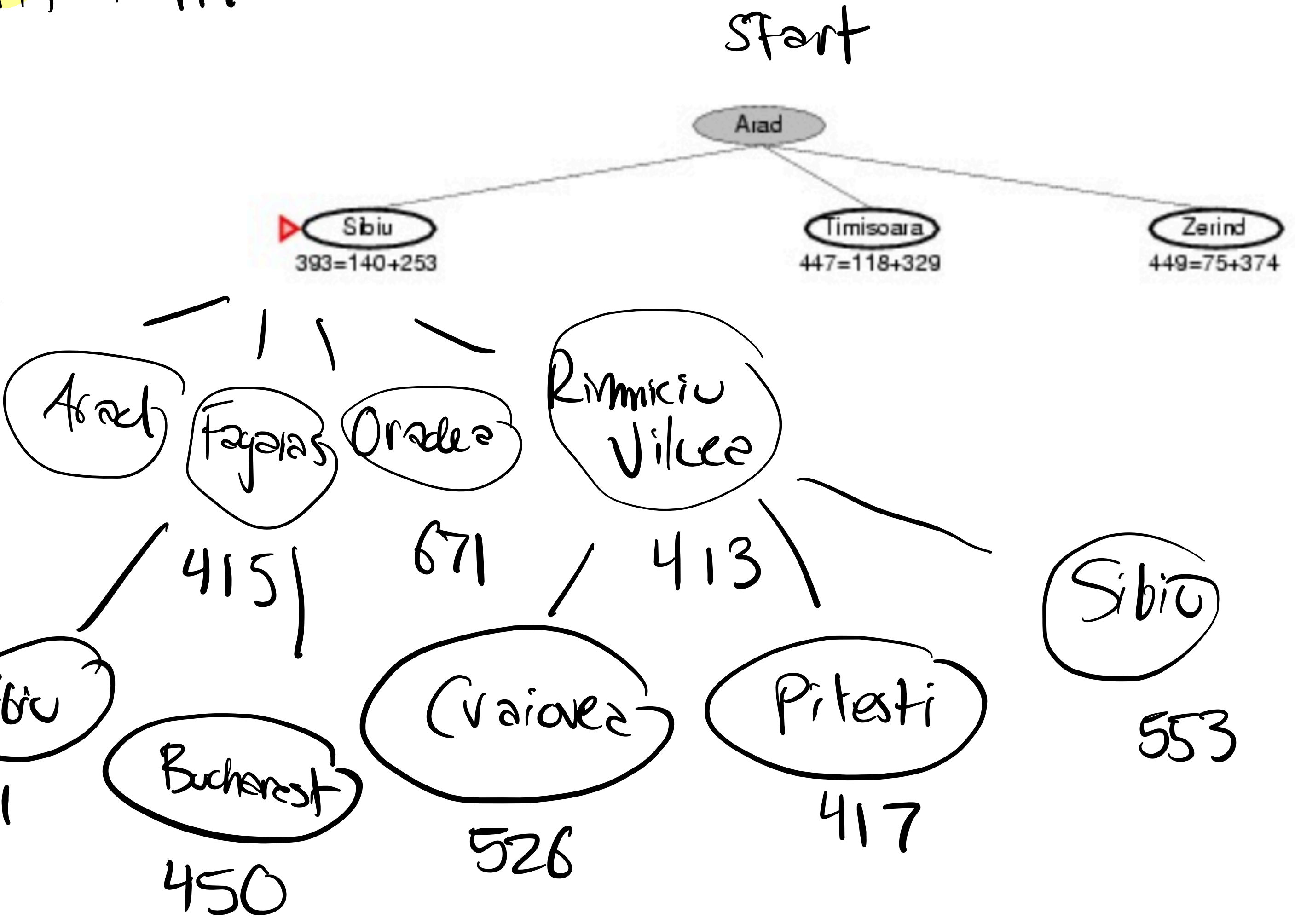

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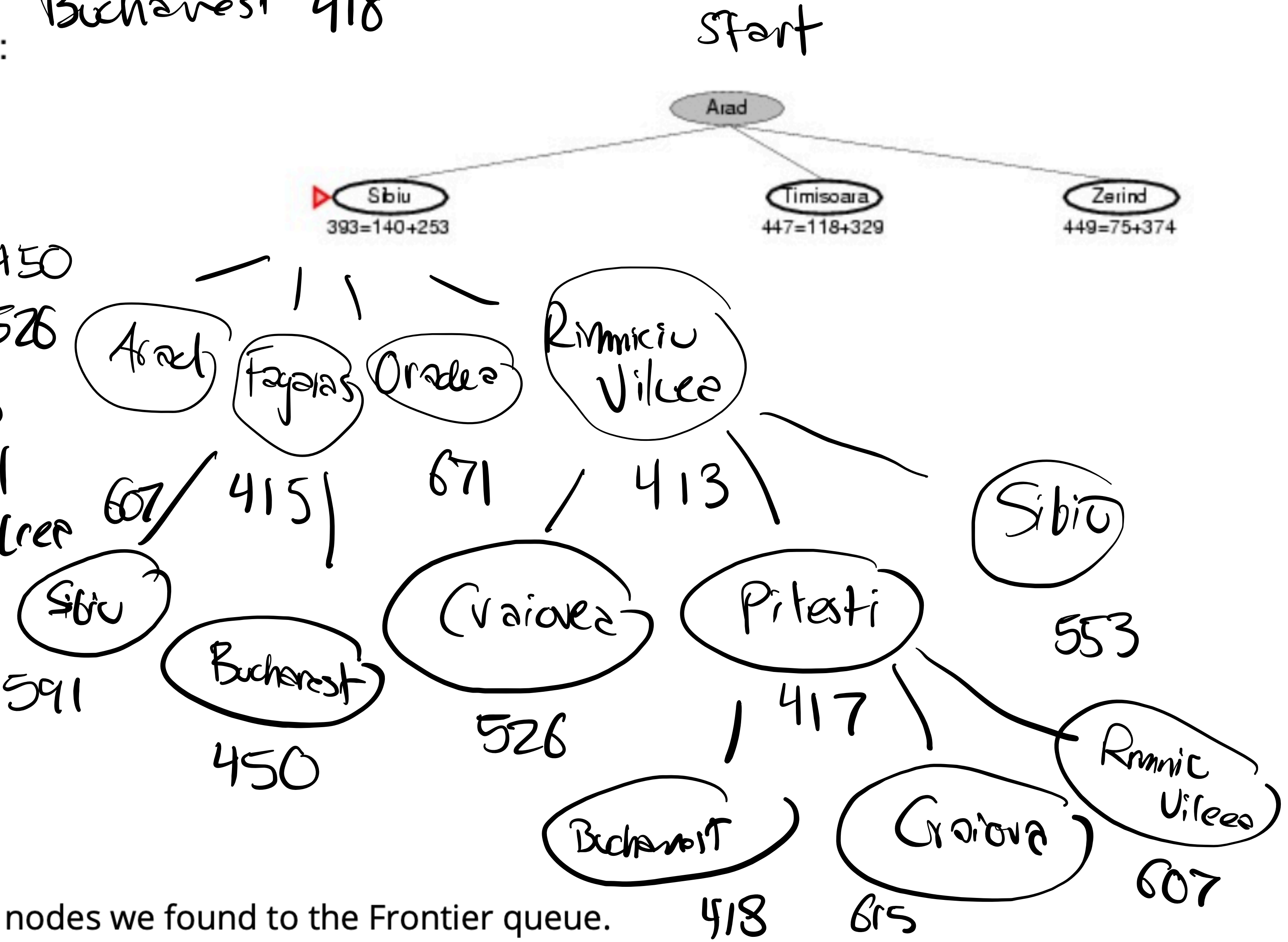

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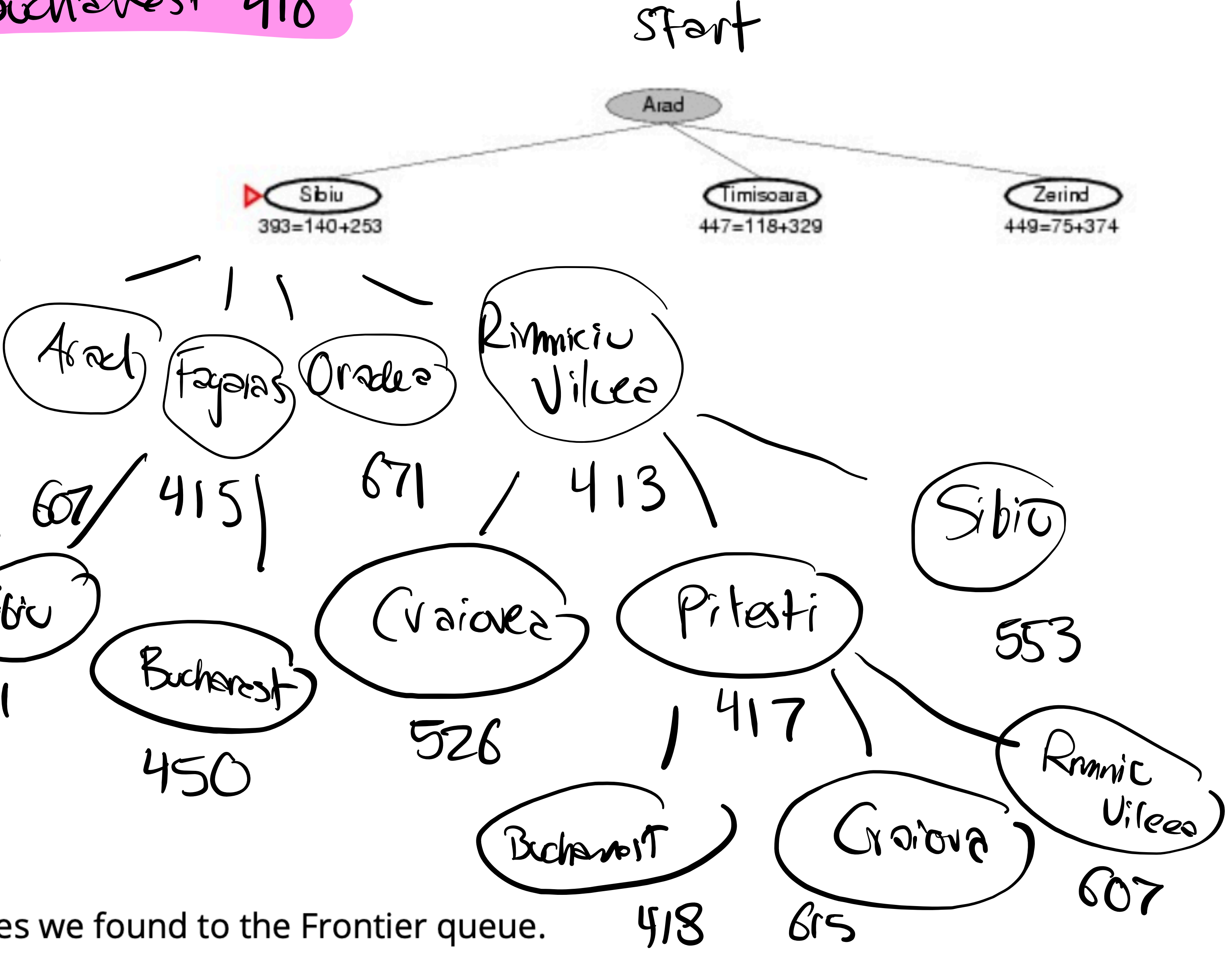

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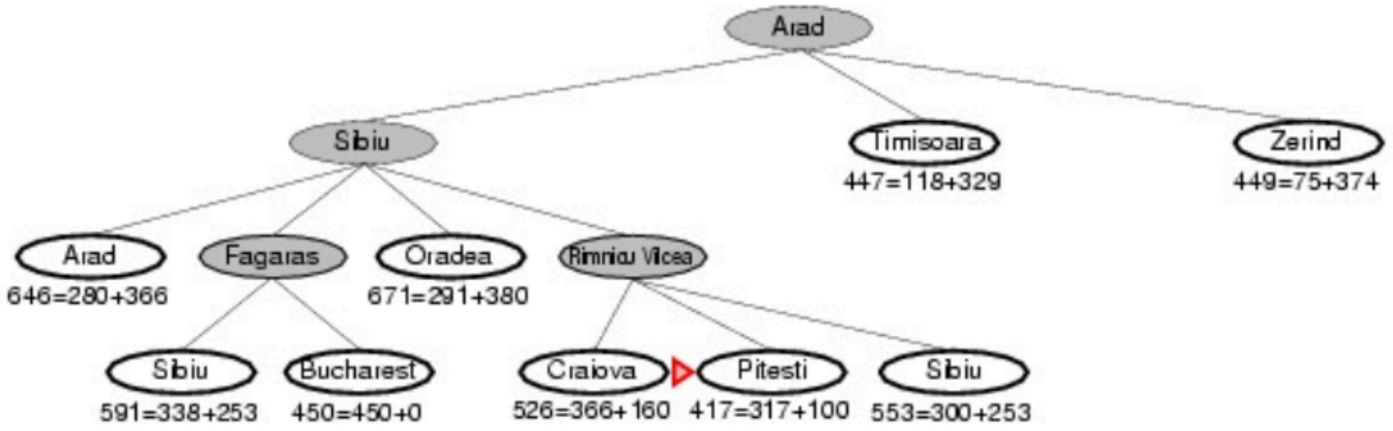

When we expand Fagaras, we find Bucharest, but we're not done. The algorithm doesn't end until we "expand" the goal node – it has to be at the top of the Frontier queue.

A* search example

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Bucharest 450

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Sibiu 591

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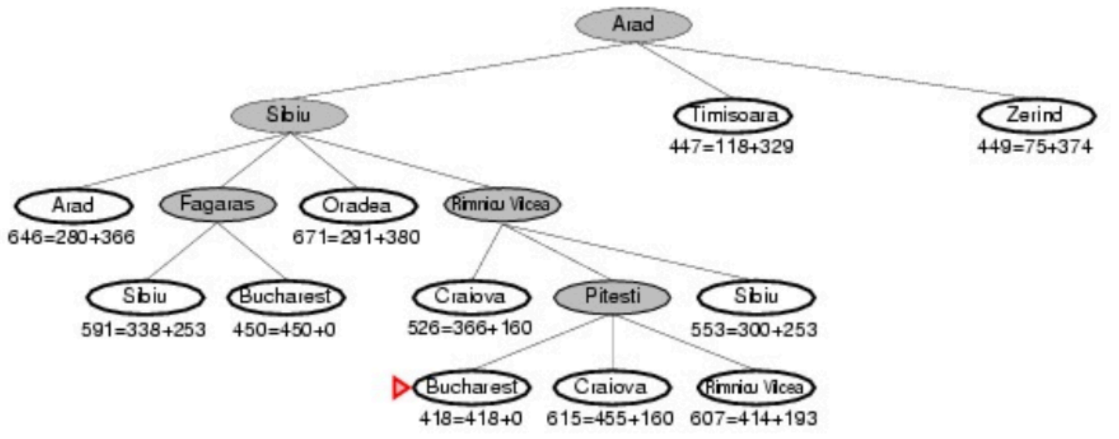


Note that we just found a better value for Bucharest!

Now we expand this better value for Bucharest since it's at the top of the queue.

We're done and we know the value found is optimal!

Adversarial Search

Search

So far, we have only considered one-player games.

What happens when we add another player?

Multiplayer Games

In competitive multiplayer games, we have to consider our opponent’s possible actions, as well as our own.

We call this **adversarial search.**

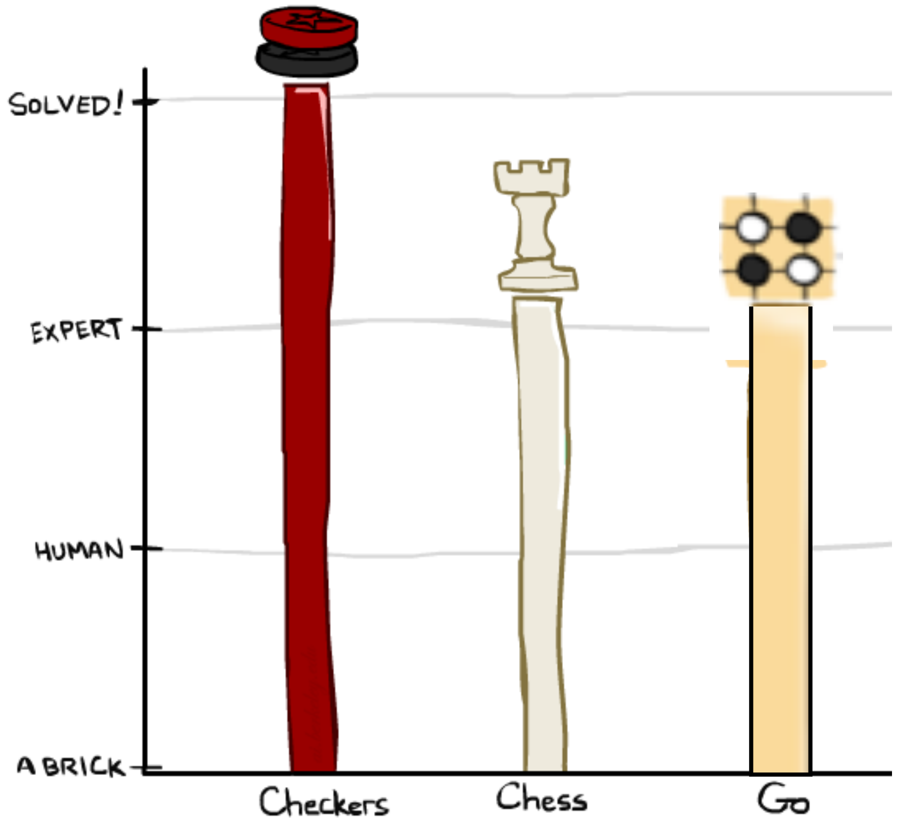
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go: 2016: Alpha GO defeats human champion. Uses Monte Carlo Tree Search + neural network to learn evaluation function.**

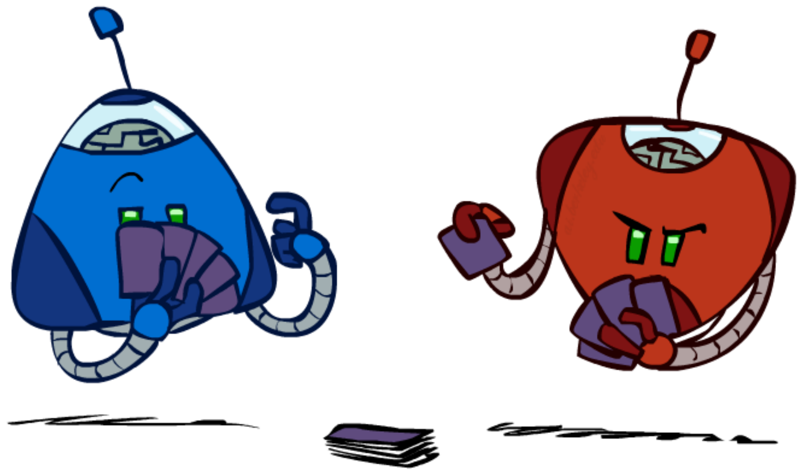
- **Go + Chess + Shogi: 2017: Alpha Zero learns all 3 games using reinforcement learning to play against itself.**



Types of Games

- Many different kinds of games!

- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?

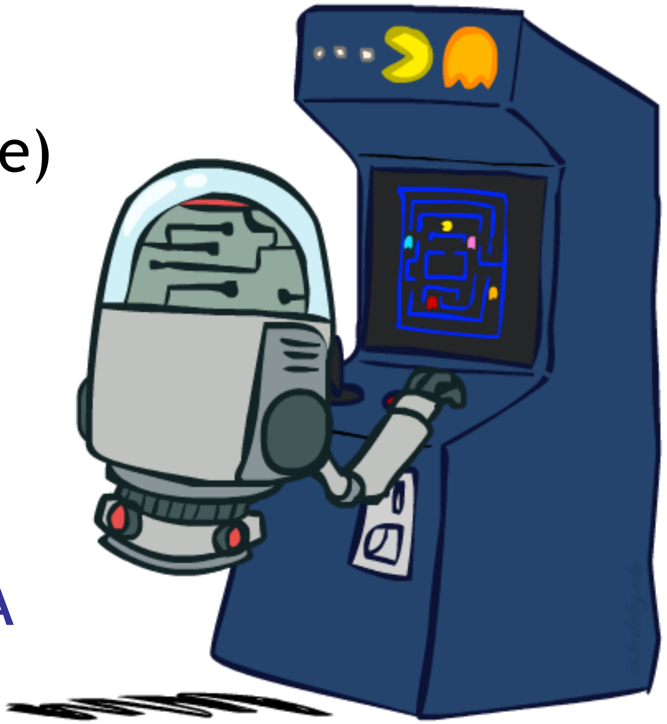


- Want algorithms for calculating a strategy (policy) which recommends a move from each state

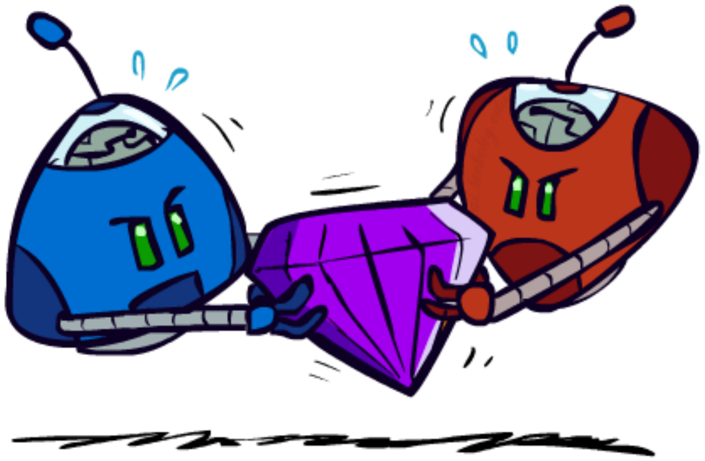
Deterministic Games

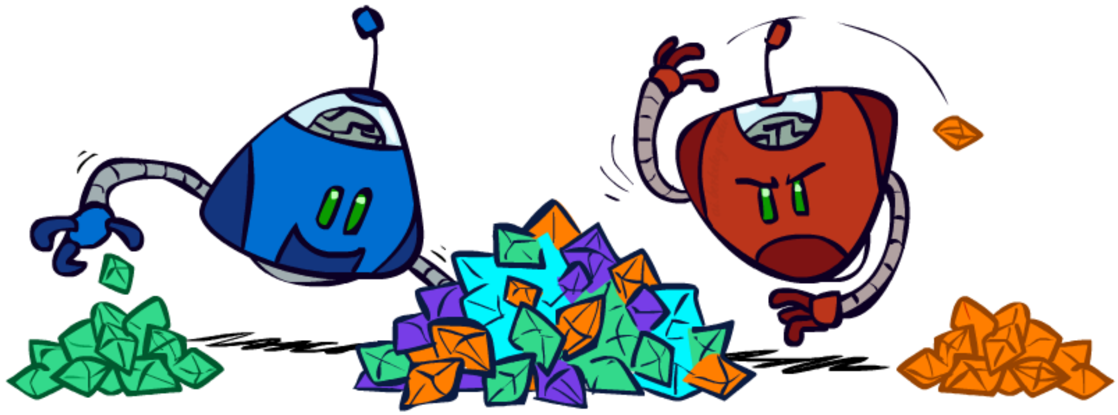
- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R

- Solution for a player is a policy: S $\rightarrow$ A



Zero-Sum Games

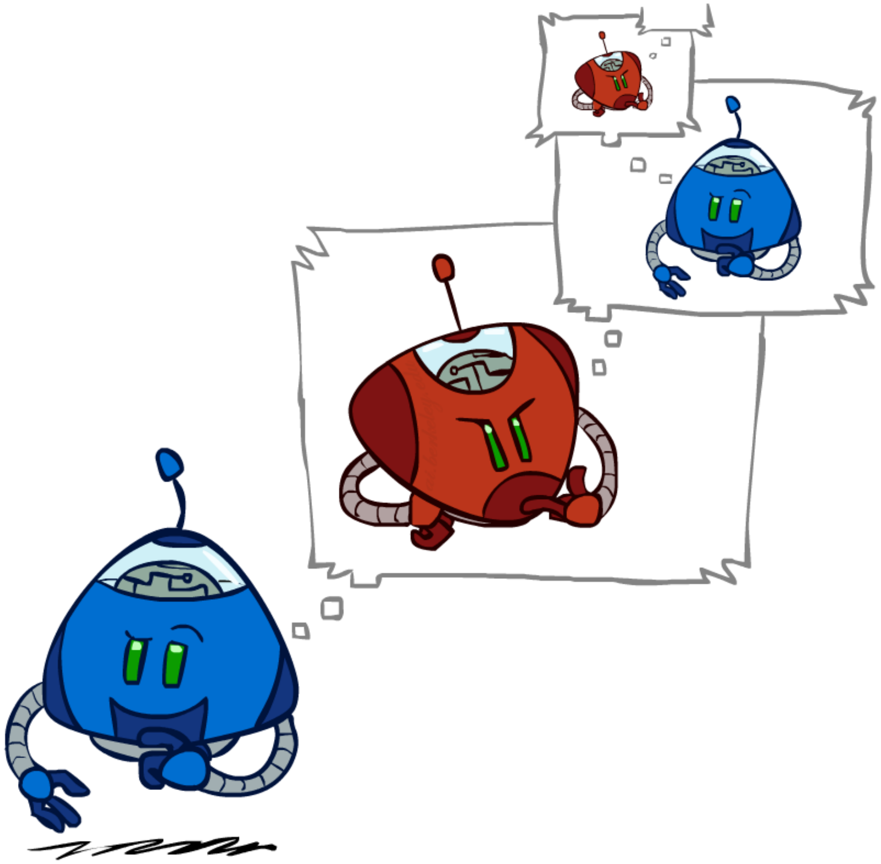




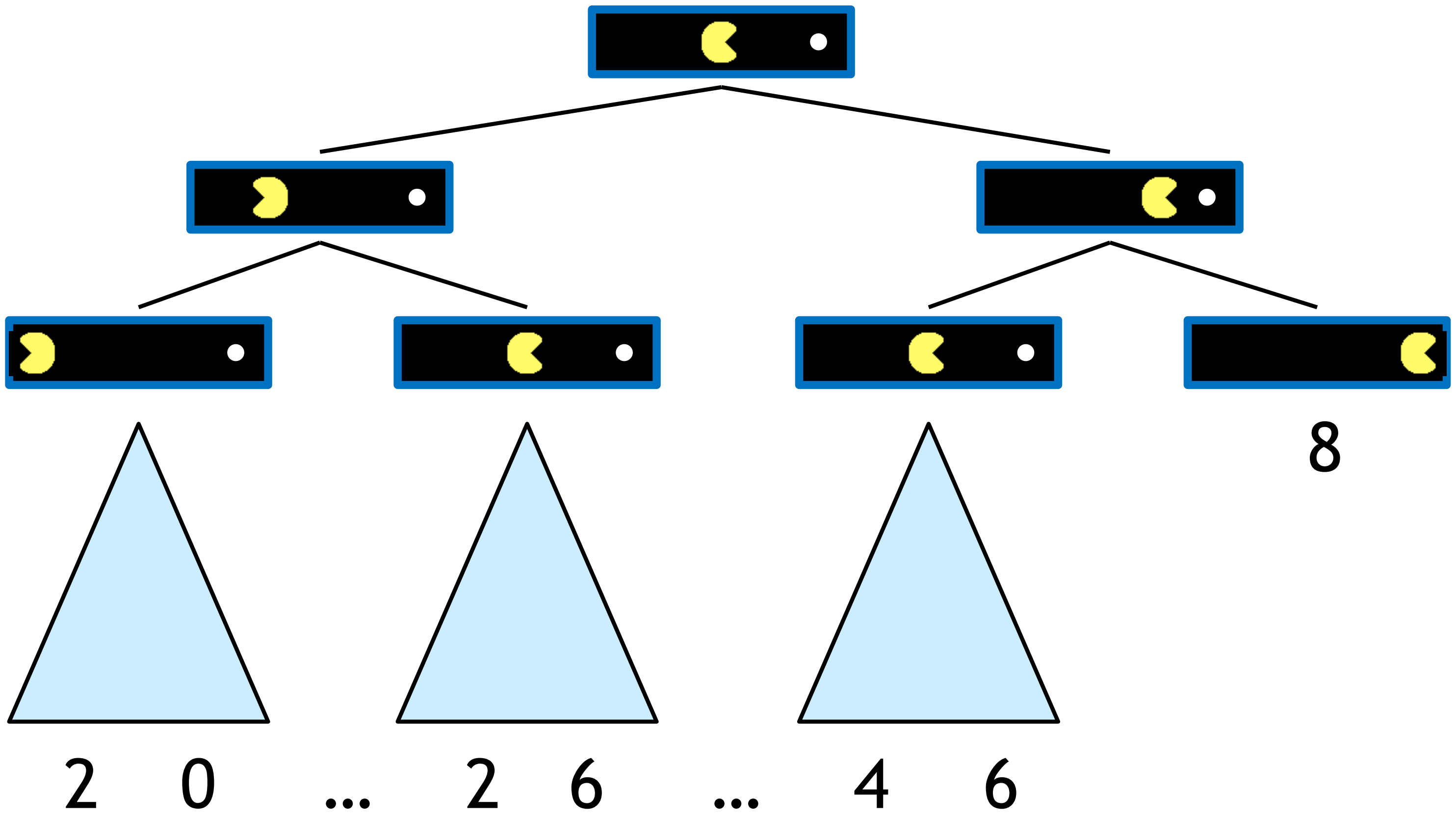
- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition

- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible

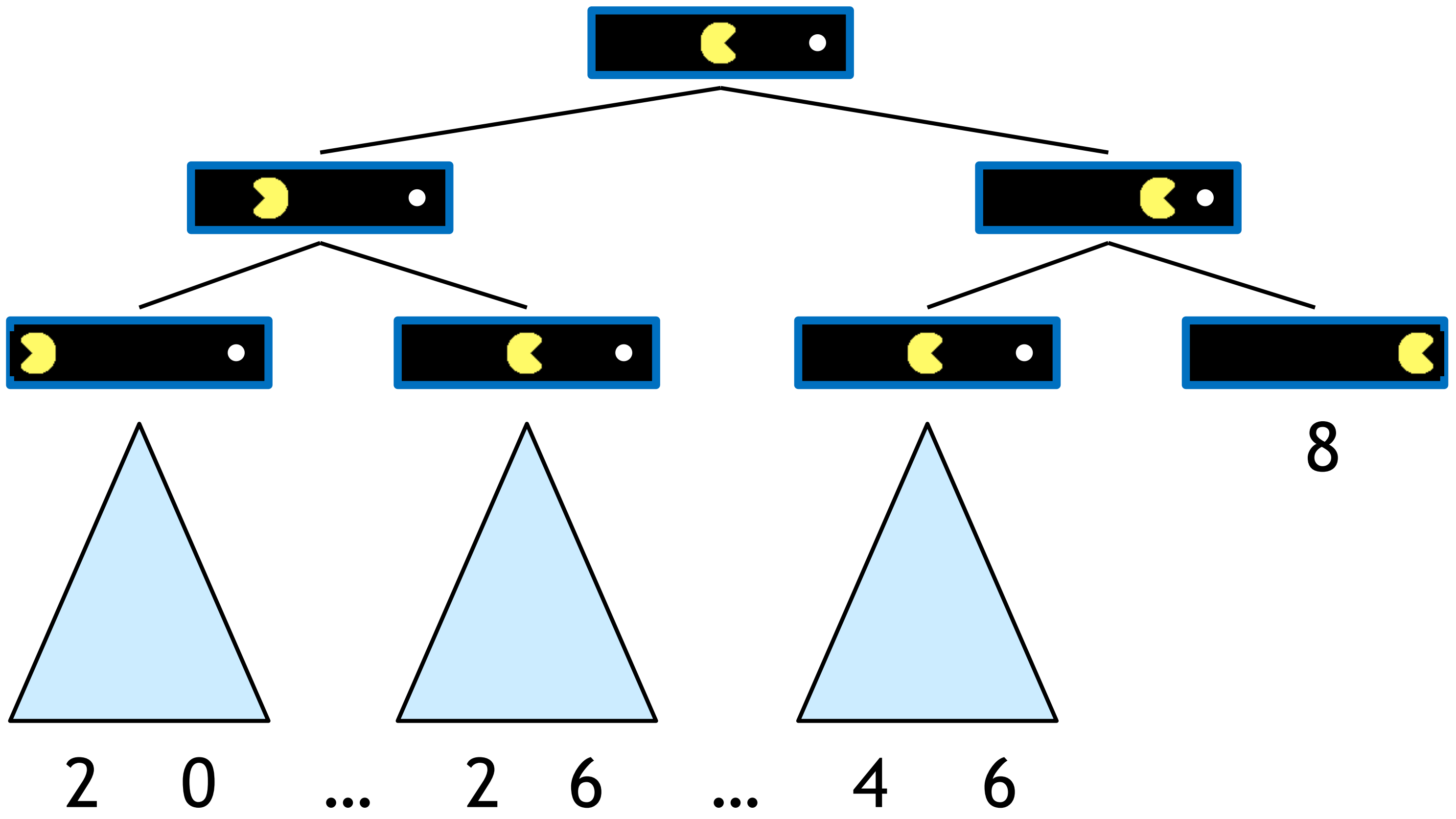
Adversarial Search



Single-Agent Trees



Value of a State

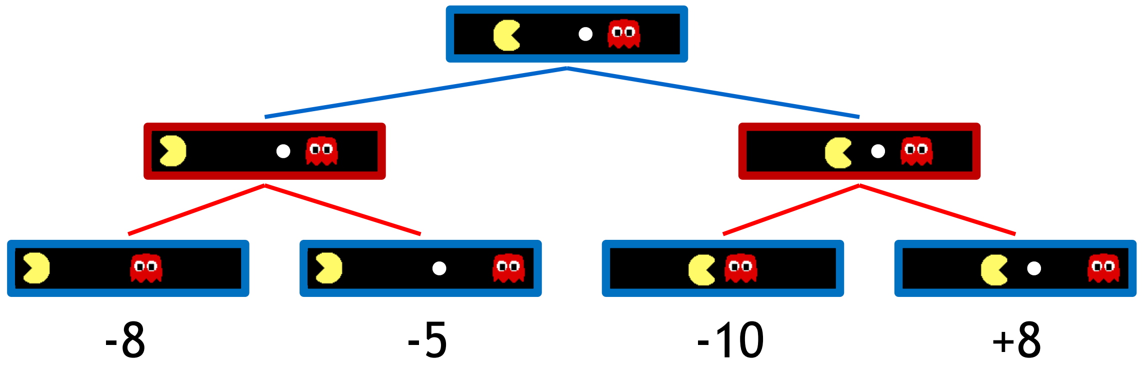


Adversarial Game Trees

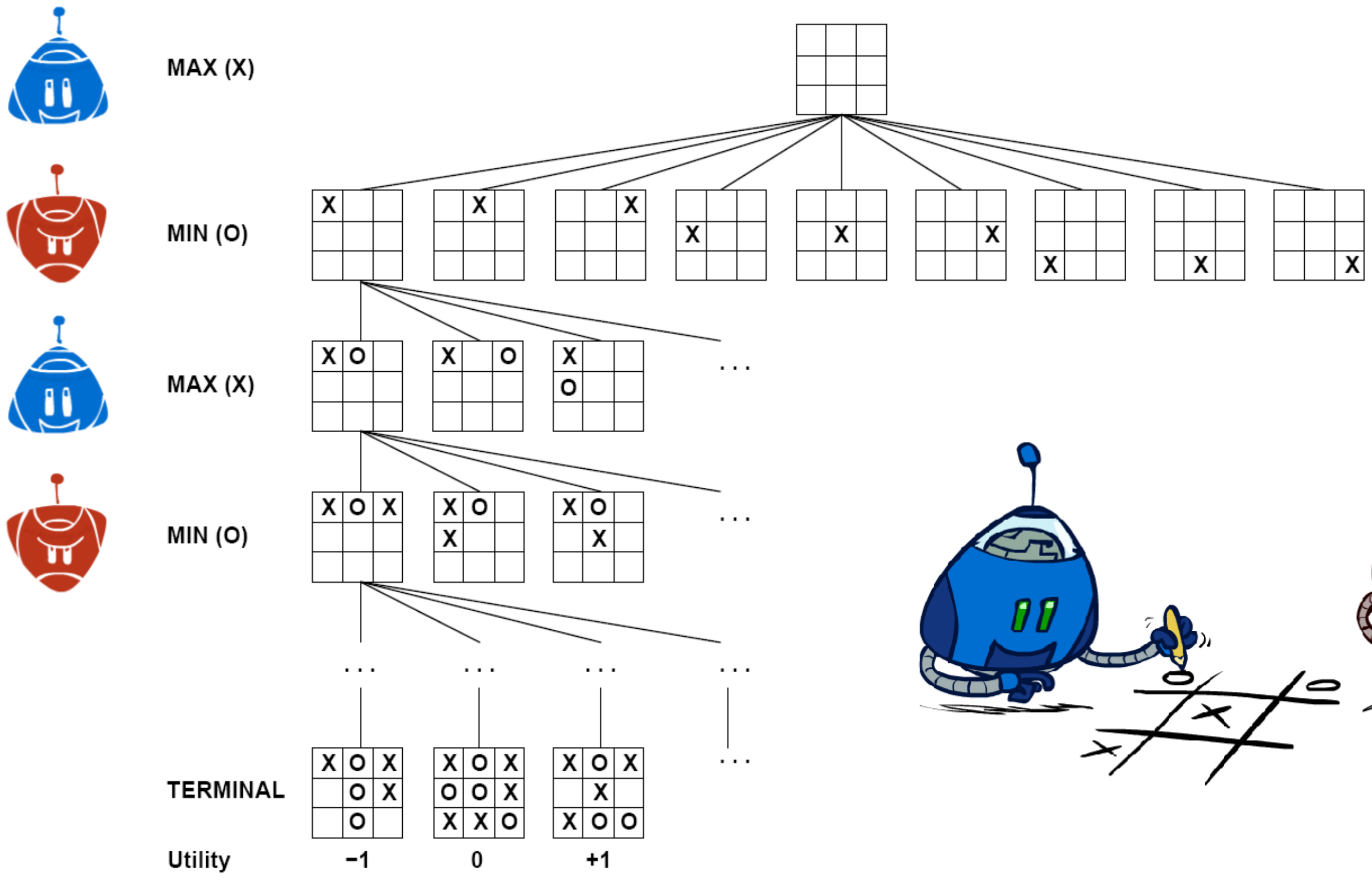
Minimax Values

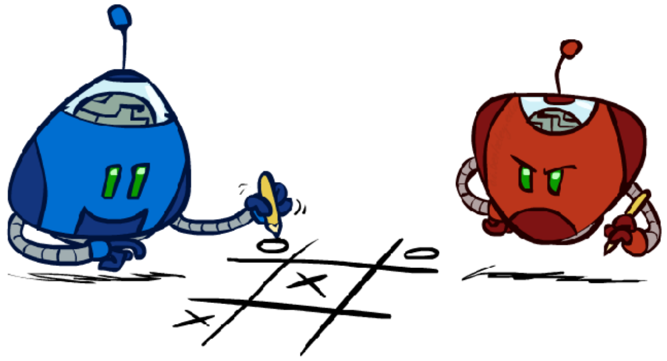
States Under Agent's Control:

States Under Opponent's Control:



Tic-Tac-Toe Game Tree



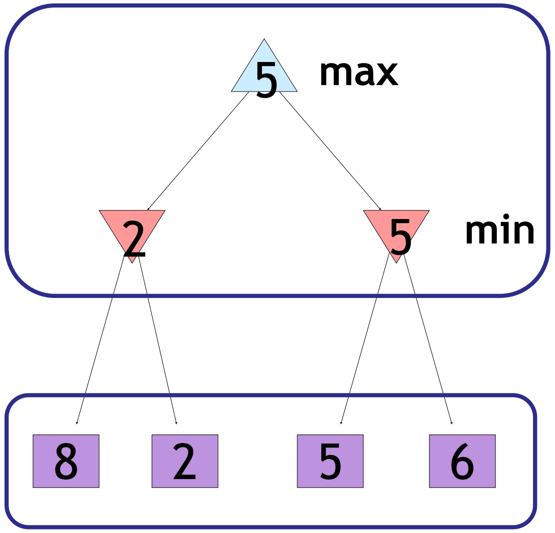


Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary

Minimax values: computed recursively



Terminal values: part of the game

Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
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    return v
```
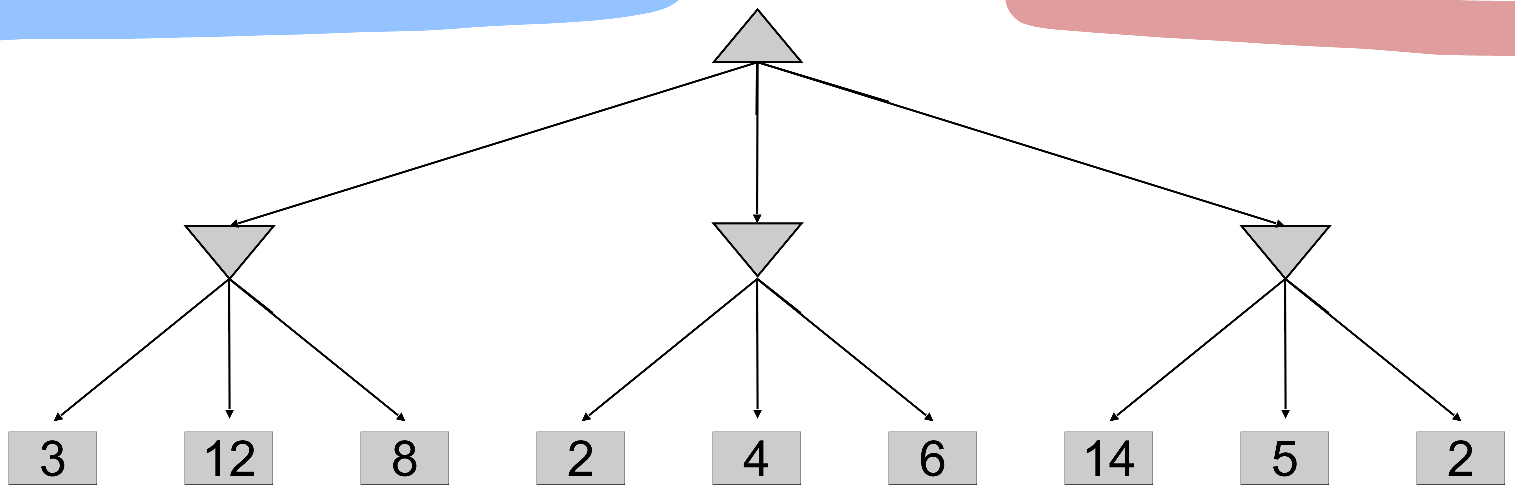
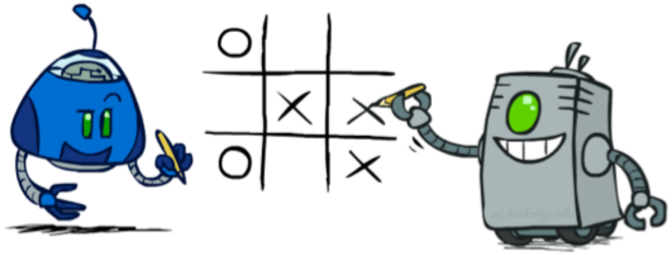
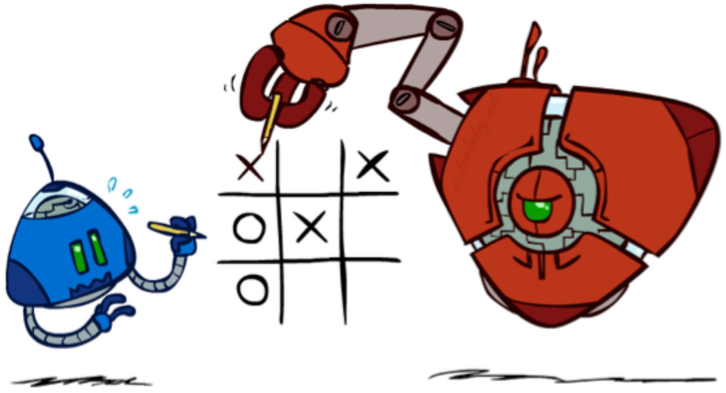
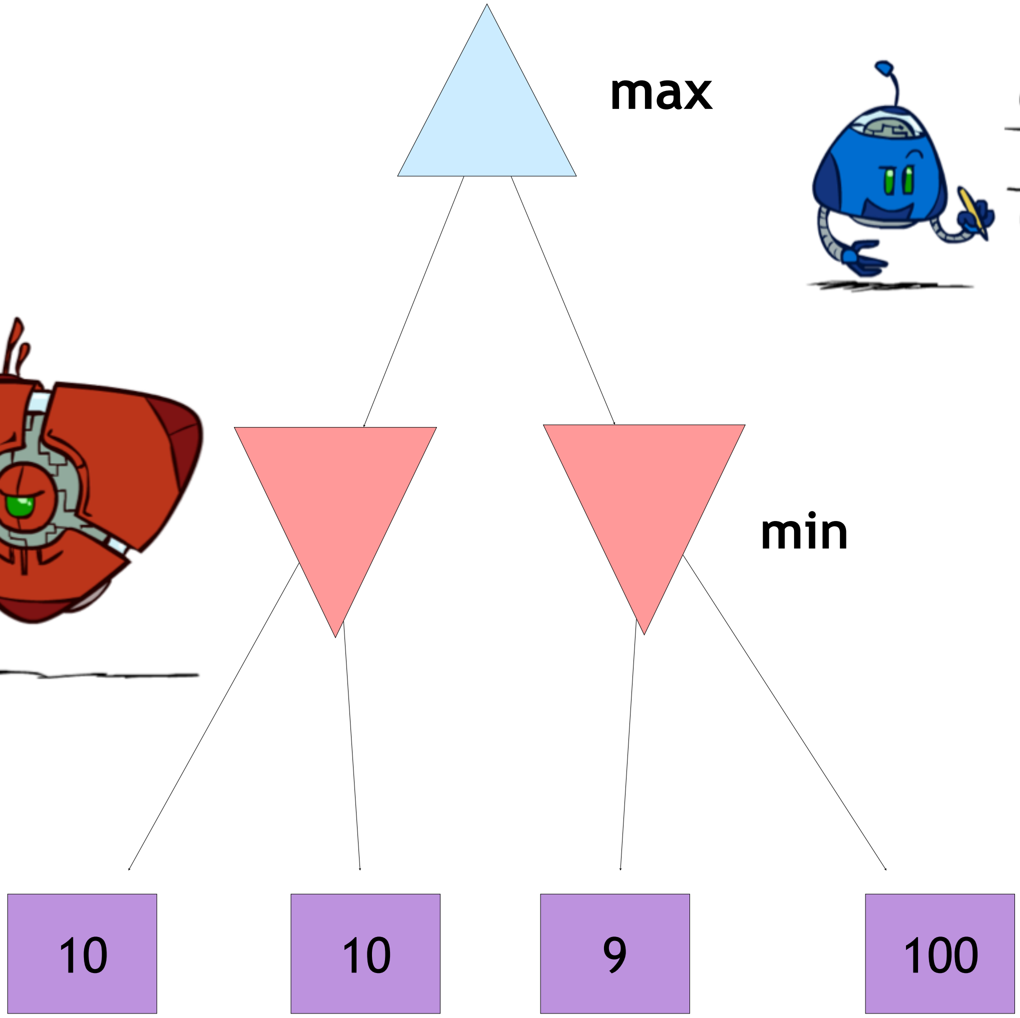

Minimax Example

Which nodes are under Max's control?

Which nodes are under Min's control?



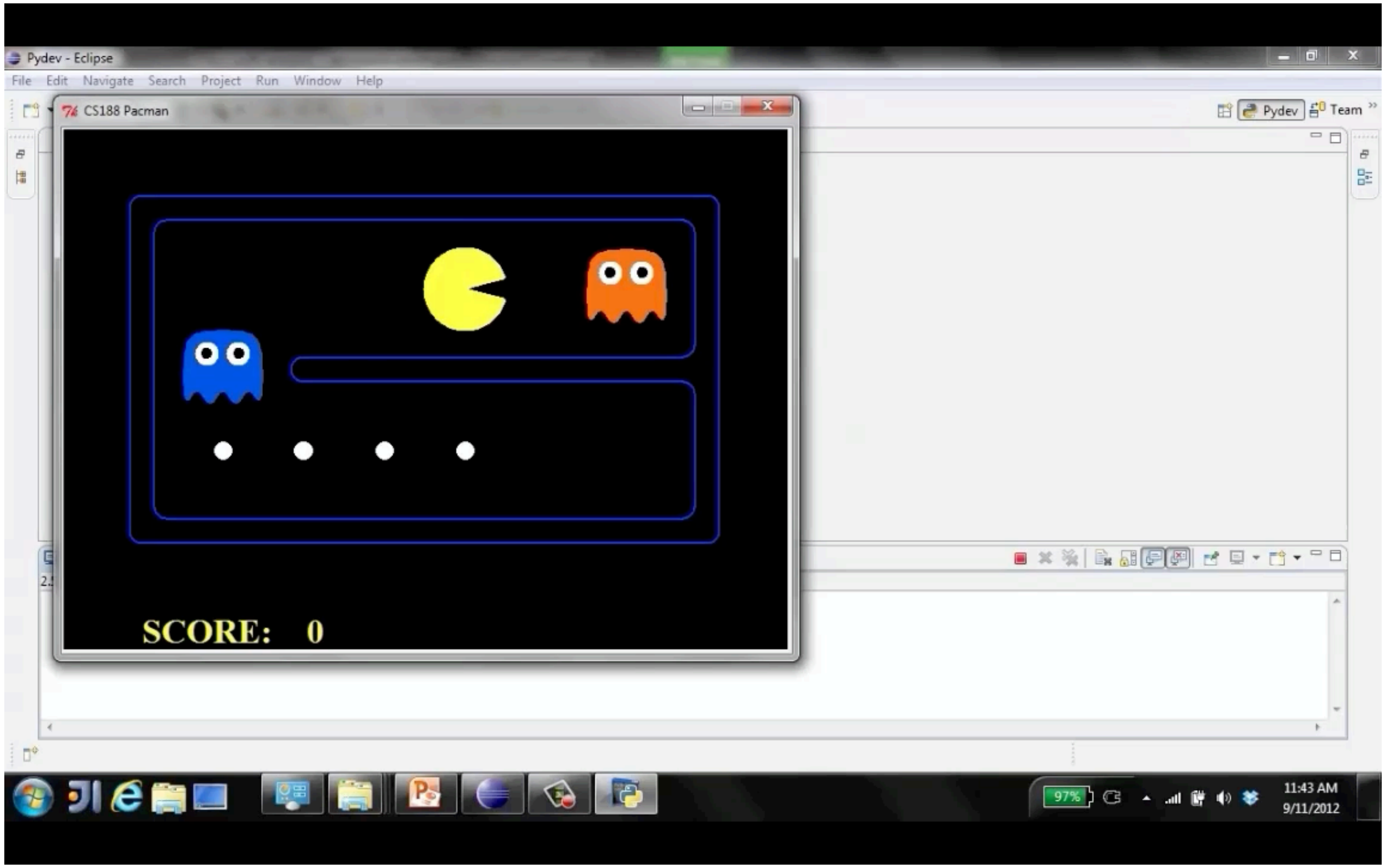
Minimax Properties

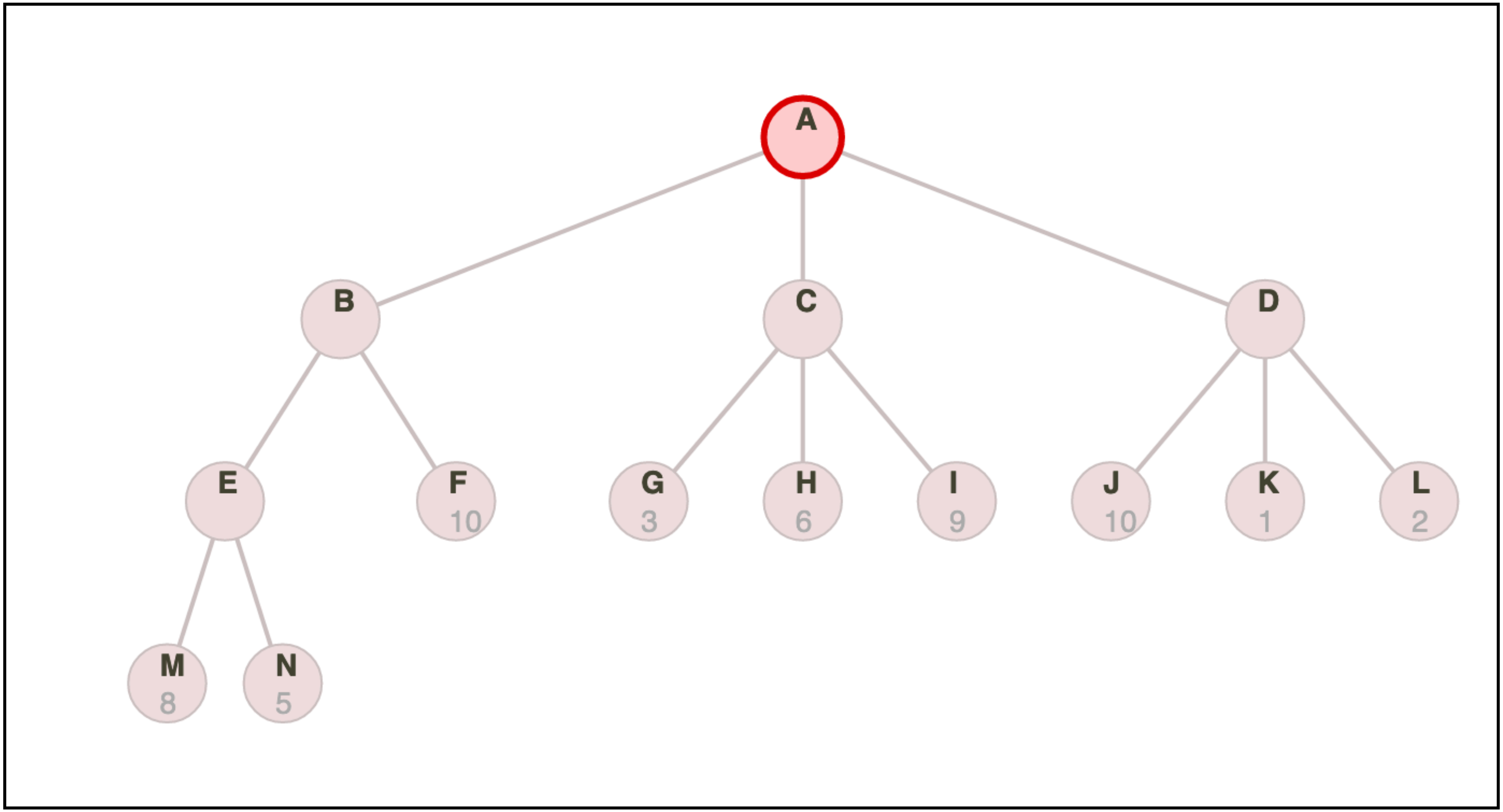
Optimal against a perfect player. Otherwise?

Video of Demo Min vs. Exp (Min)

Video of Demo Min vs. Exp (Exp)



Another Demo



Minimax Summary

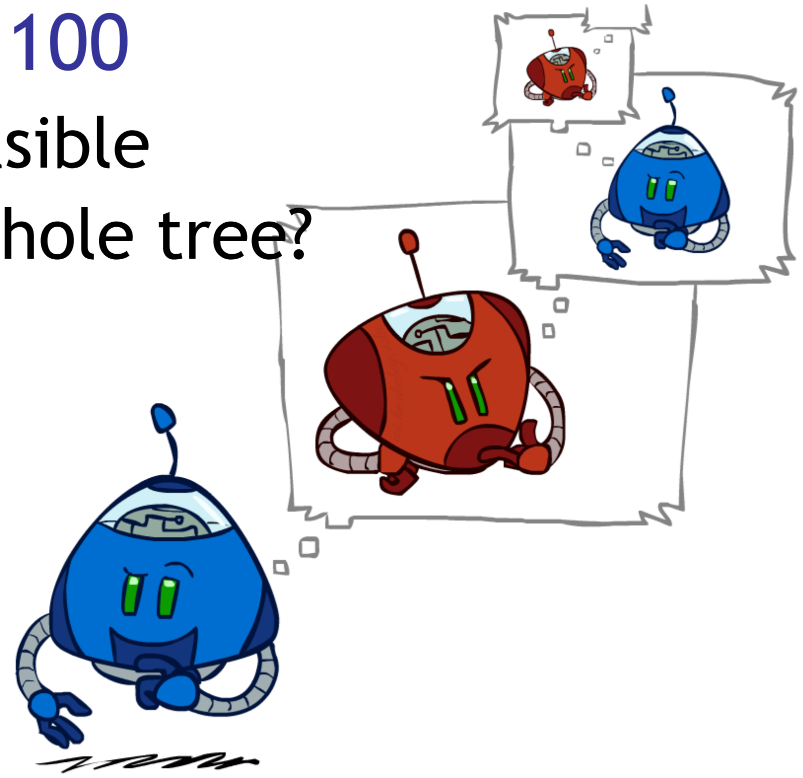
- Rank final game states by their final scores (for tic-tac-toe or chess: win, draw, loss).
- Rank intermediate game states by whose turn it is and the available moves.
  - If it's X's turn, set the rank to that of the *maximum* move available. If a move will result in a win, X should take it.
  - If it's O's turn, set the rank to that of the *minimum* move available. If a move will result in a loss, X should avoid it.

Efficiency

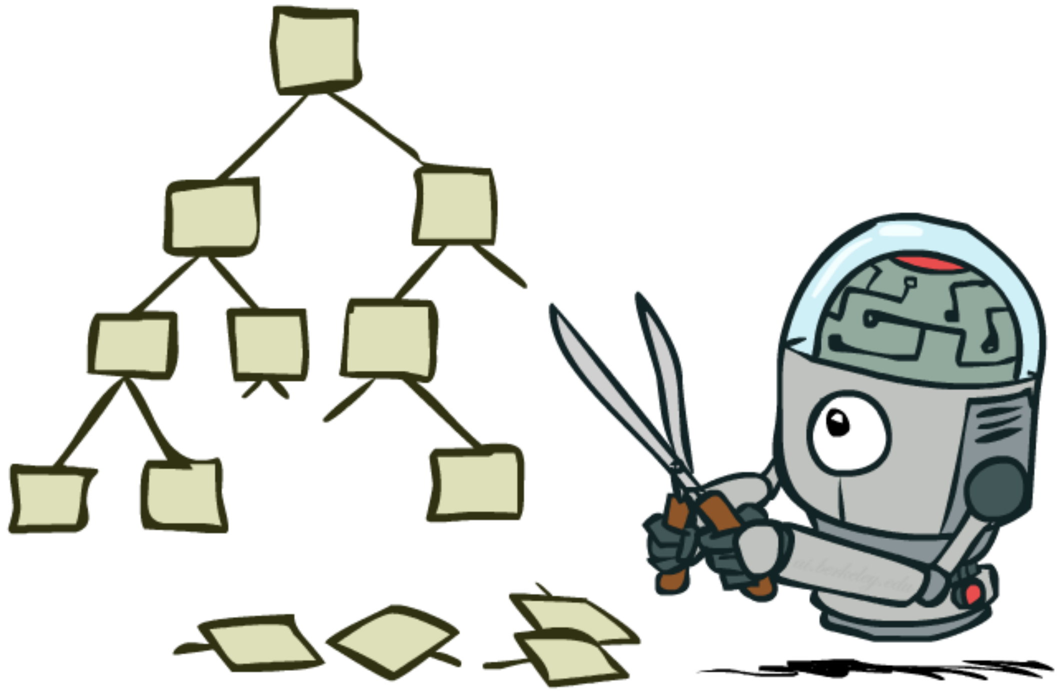
Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Game Tree Pruning



Pruning

- **Key idea**: give up on paths when you realize that they are worse than options you've already explore.
- Track the maximum score that the minimizing player (beta) can get
- Track the minimum score that the maximizing player (alpha) can get
- Whenever the maximum score that beta can get becomes less than the minimum score that alpha can get, the maximizing player can stop searching down this path, because it will never be reached.

Minimax Example

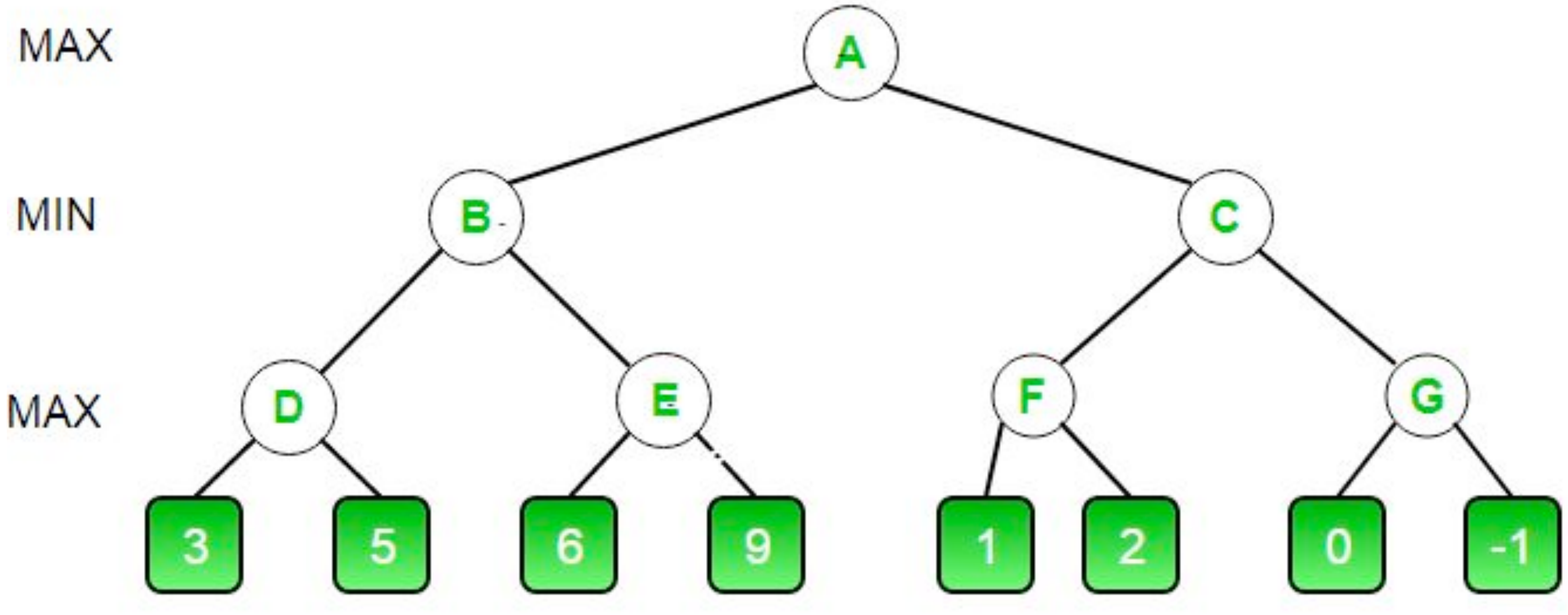
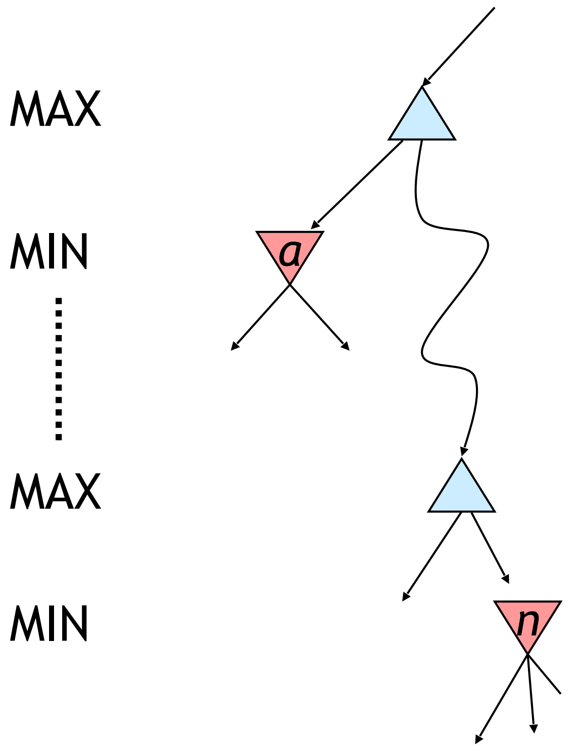


Image from www.geeksforgeeks.org/minimax-algorithm-in-game-theory-set-4-alpha-beta-pruning

Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If *n* becomes worse than *a*, MAX will avoid it, so we can stop considering *n*'s other children (it's already bad enough that it won't be played)
- MAX version is symmetric



Alpha-Beta Implementation

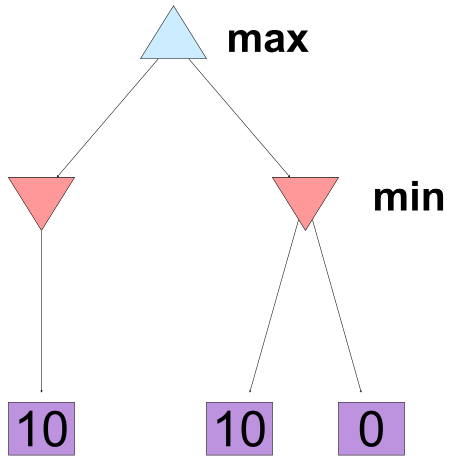
α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

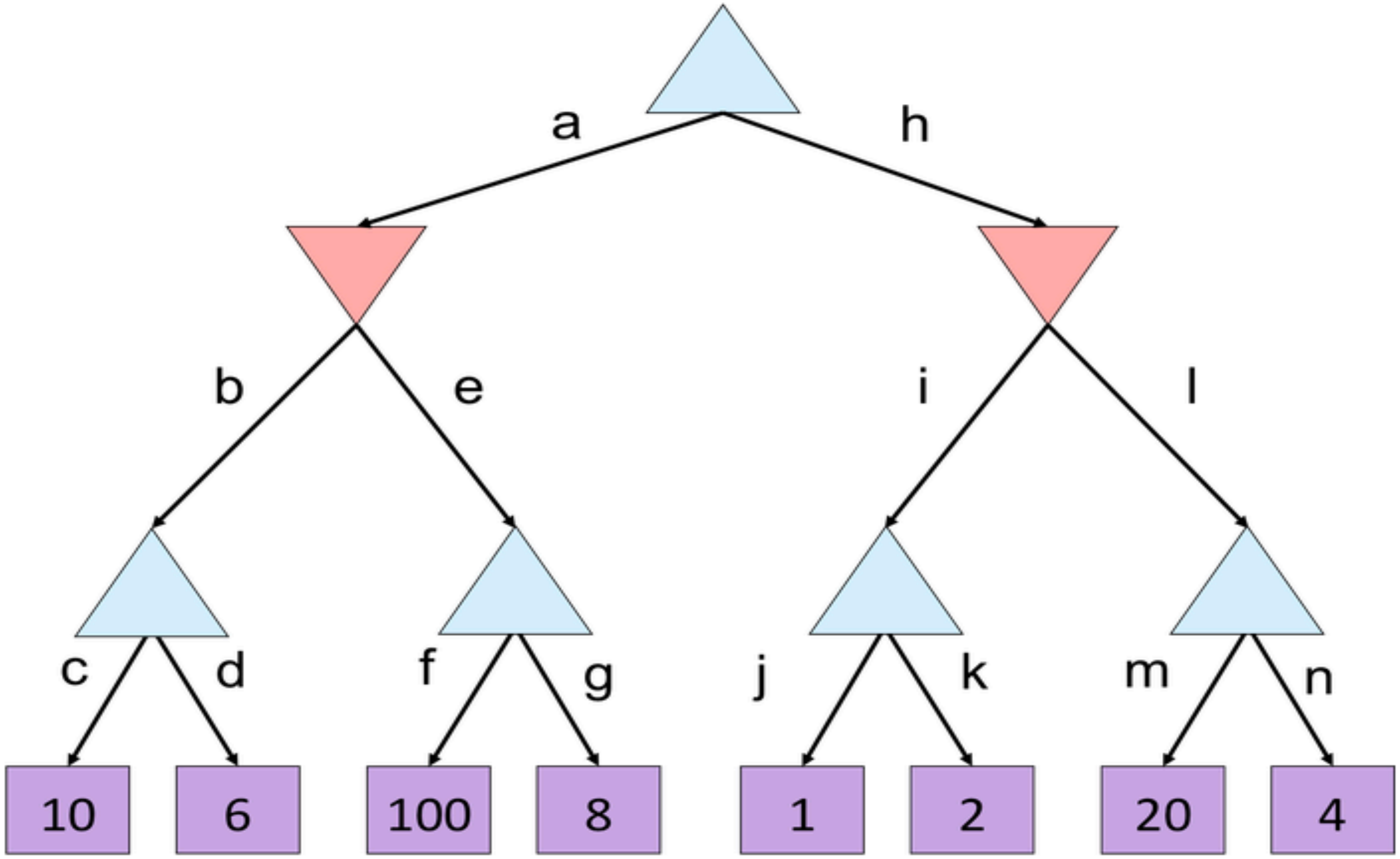
```
def min-value(state, α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

Alpha-Beta Pruning Properties

- This pruning has no effect on minimax value computed for the root!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection
- Good child ordering improves effectiveness of pruning
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…
- This is a simple example of metareasoning (computing about what to compute)

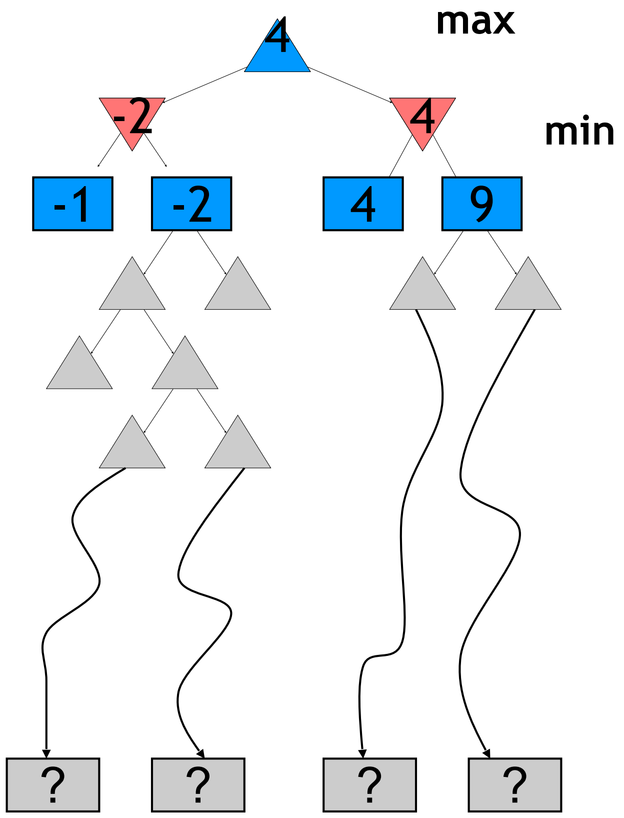


Alpha-Beta Quiz

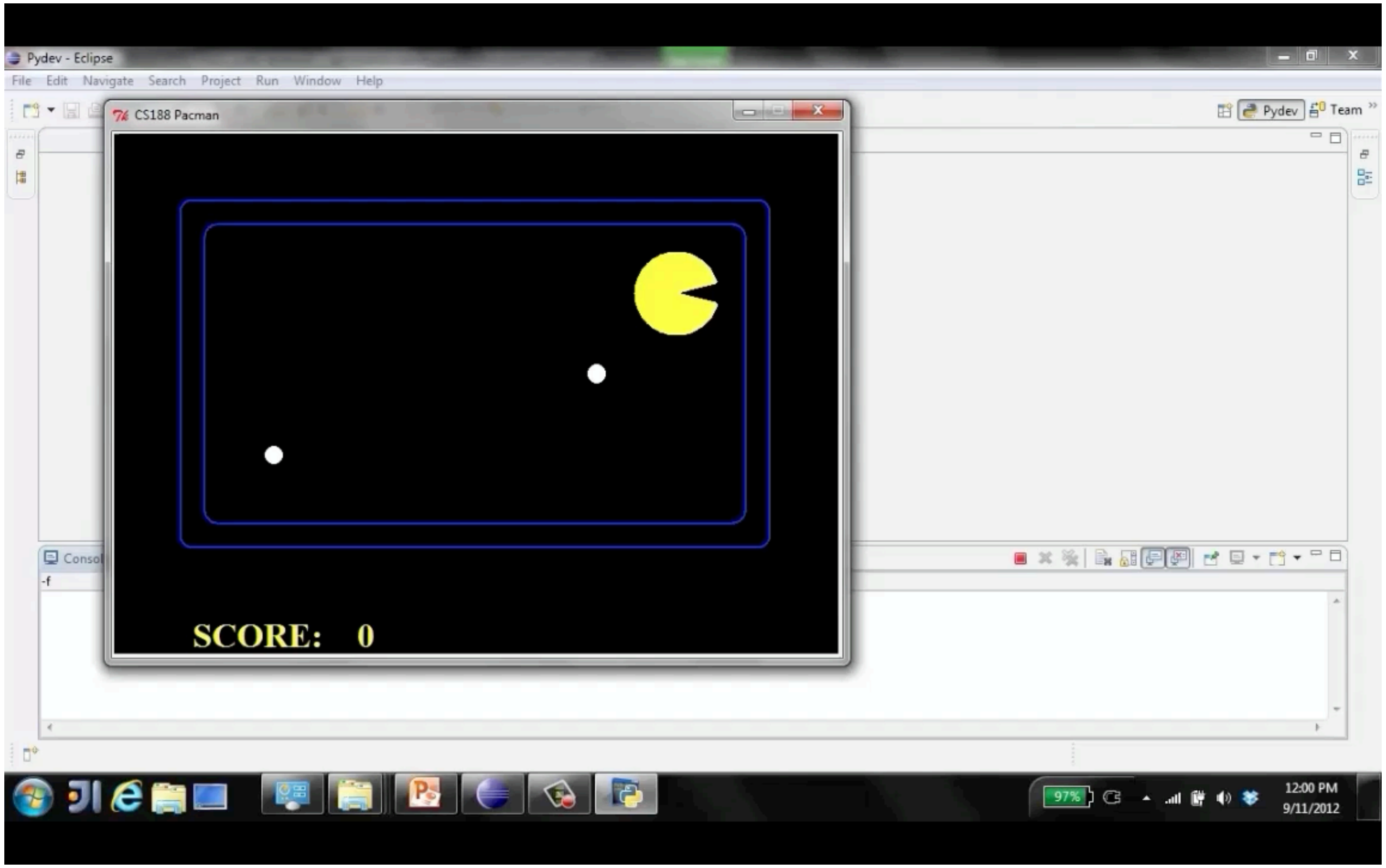


Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use iterative deepening for an anytime algorithm

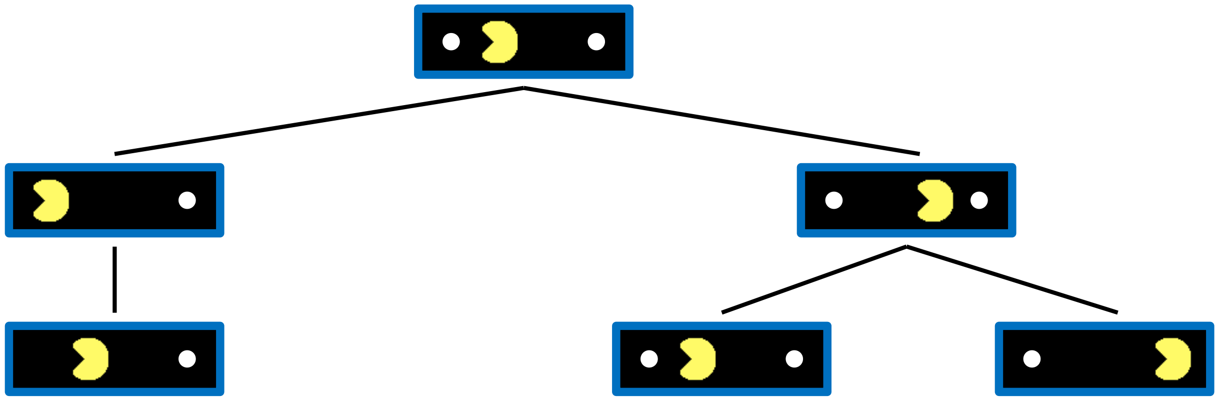


Video of Demo Thrashing (d=2)

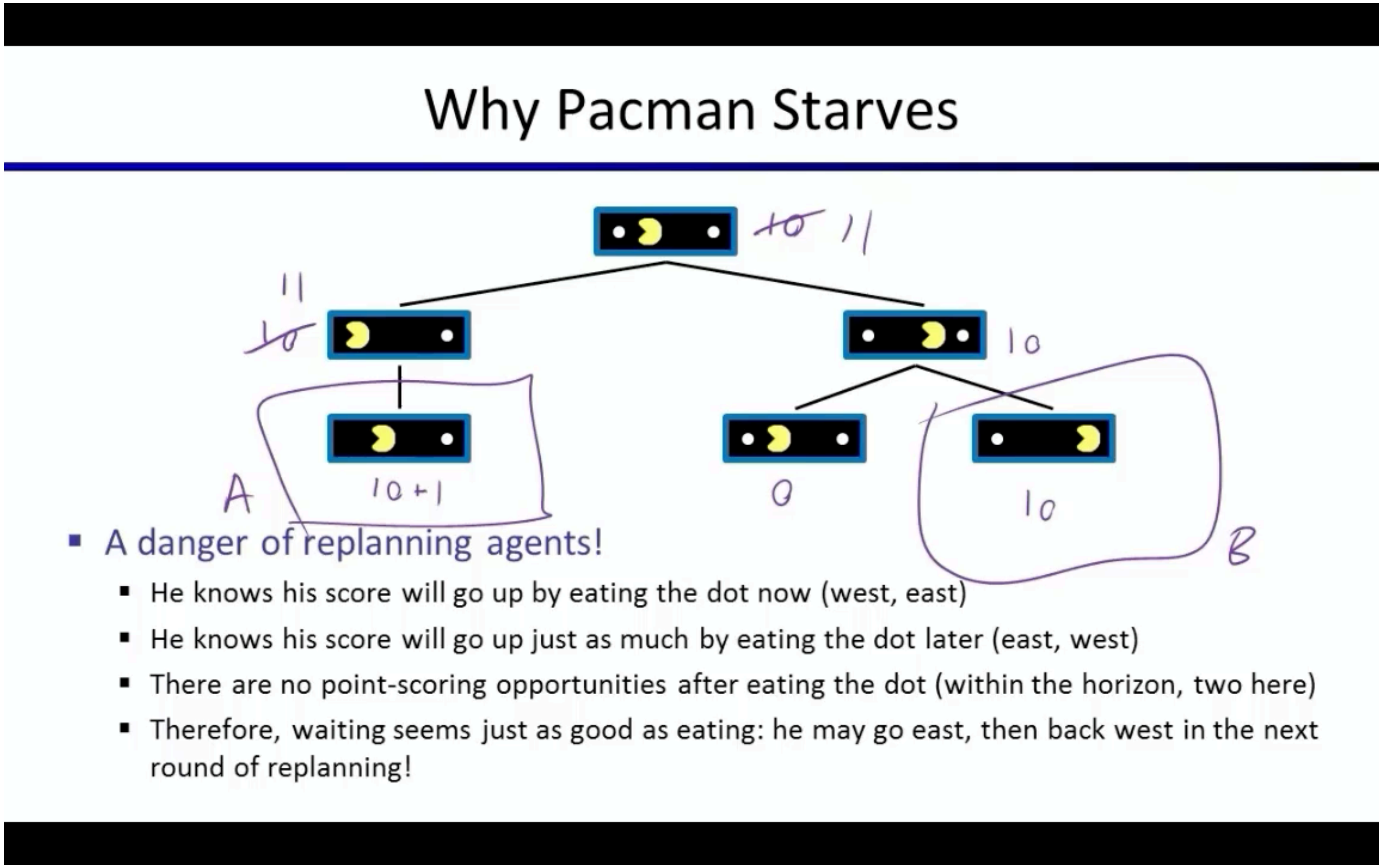


Why Pacman Starves

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!



Video of Demo Thrashing - Fixed (d=2)



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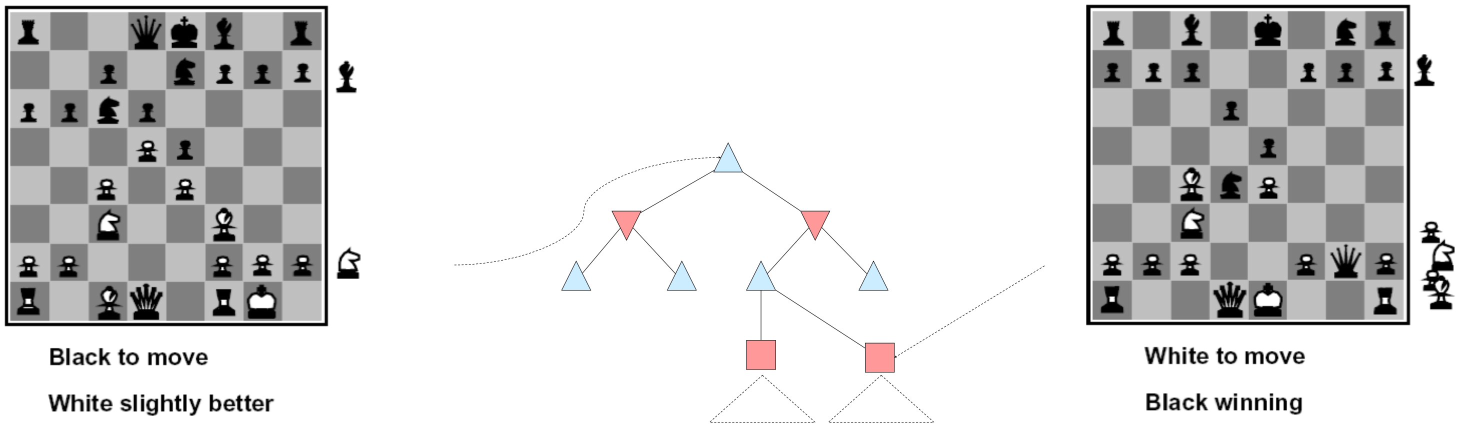
Evaluation Functions

Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search

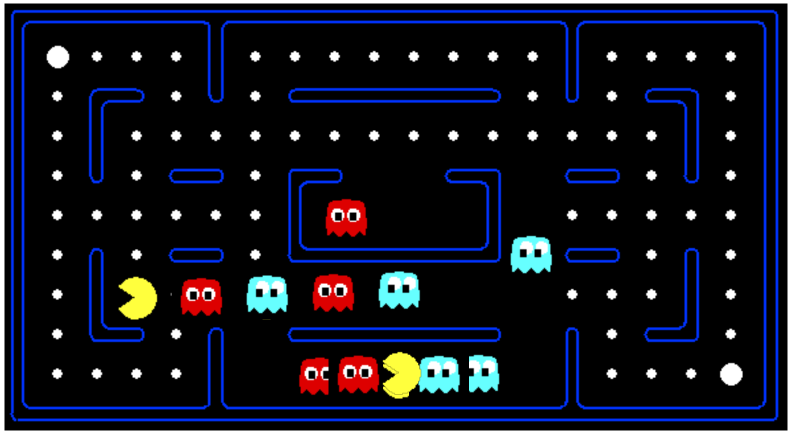


- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

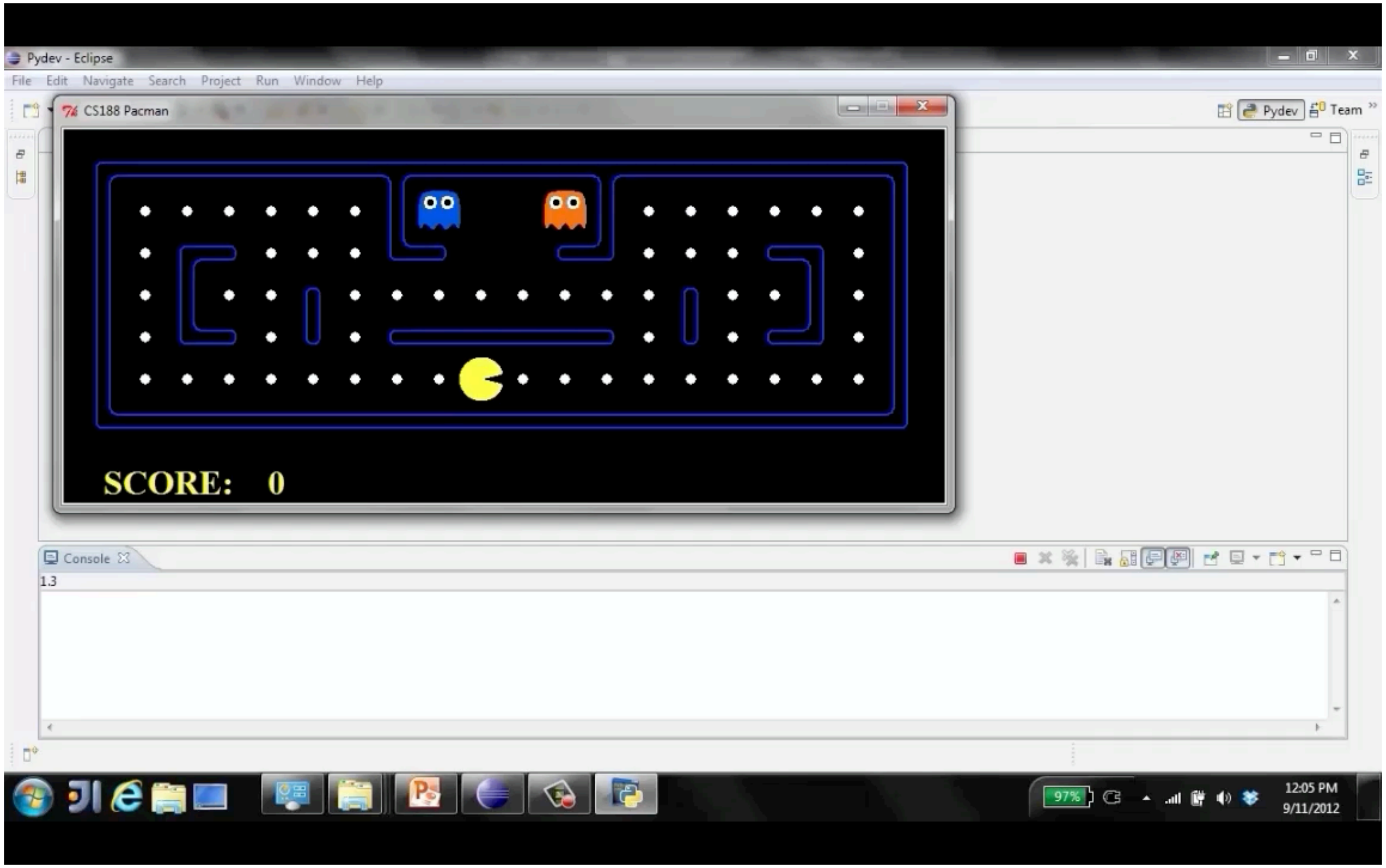
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

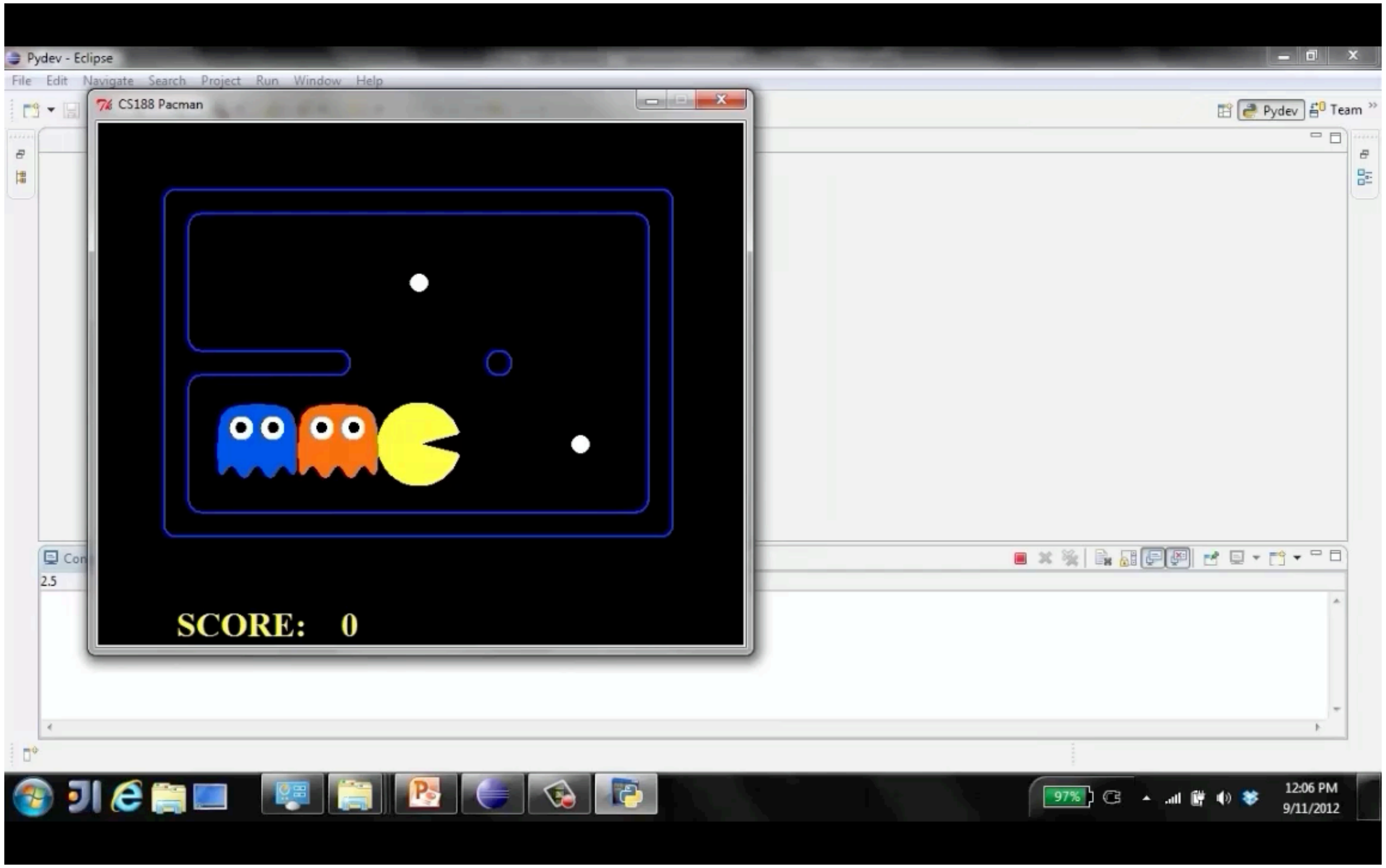
Evaluation for Pacman



Video of Demo Smart Ghosts (Coordination)

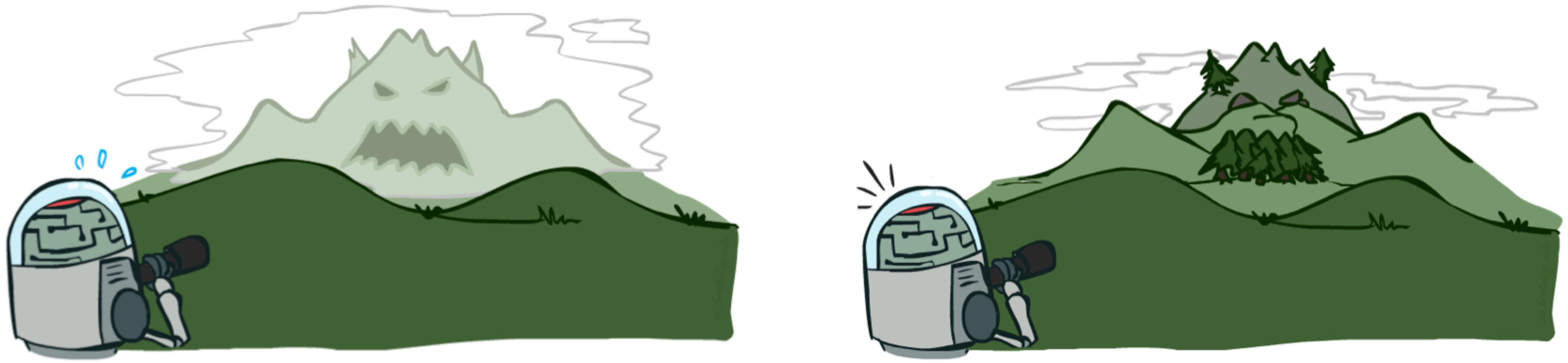


Video of Demo Smart Ghosts (Coordination) – Zoomed In

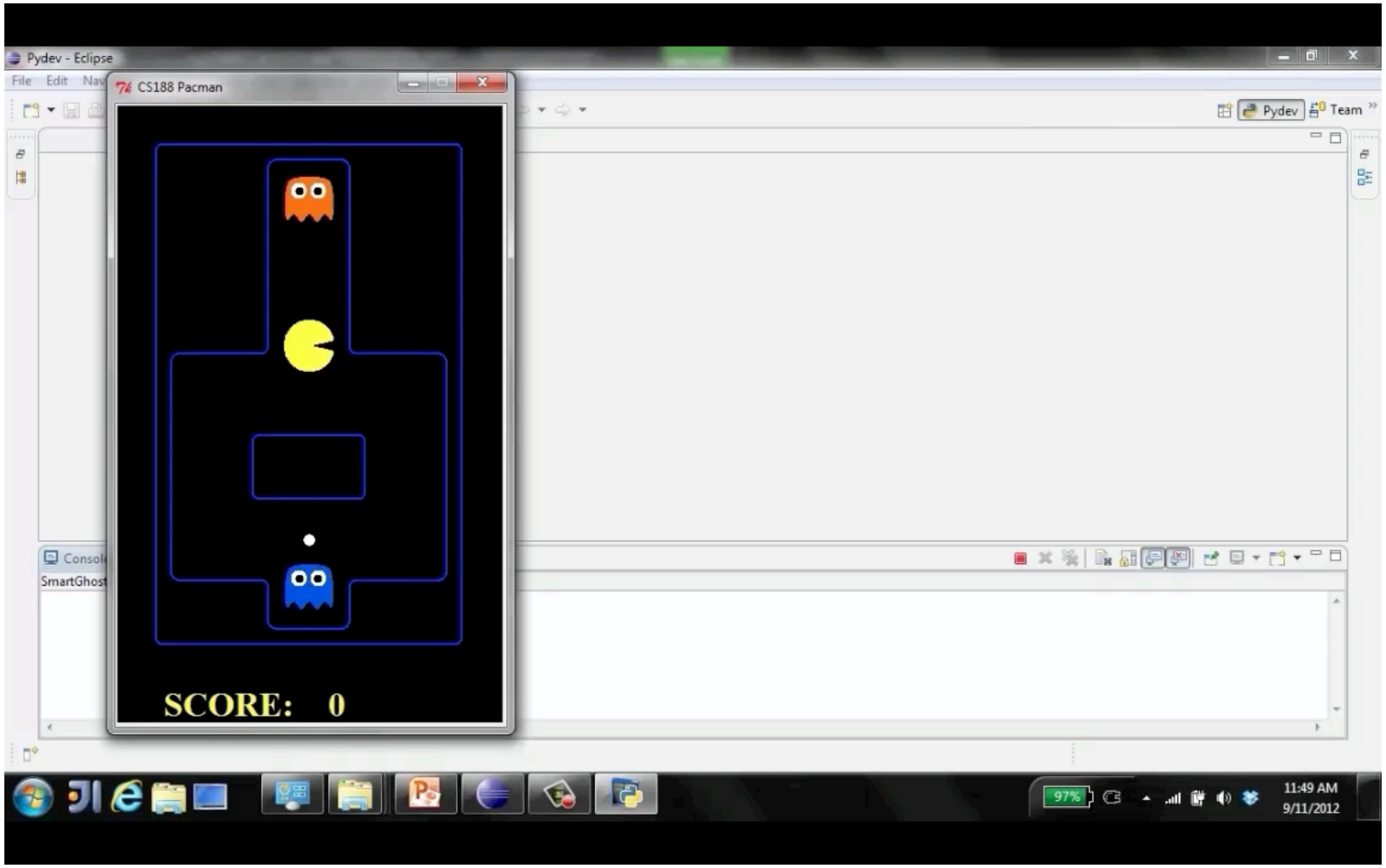


Depth Matters

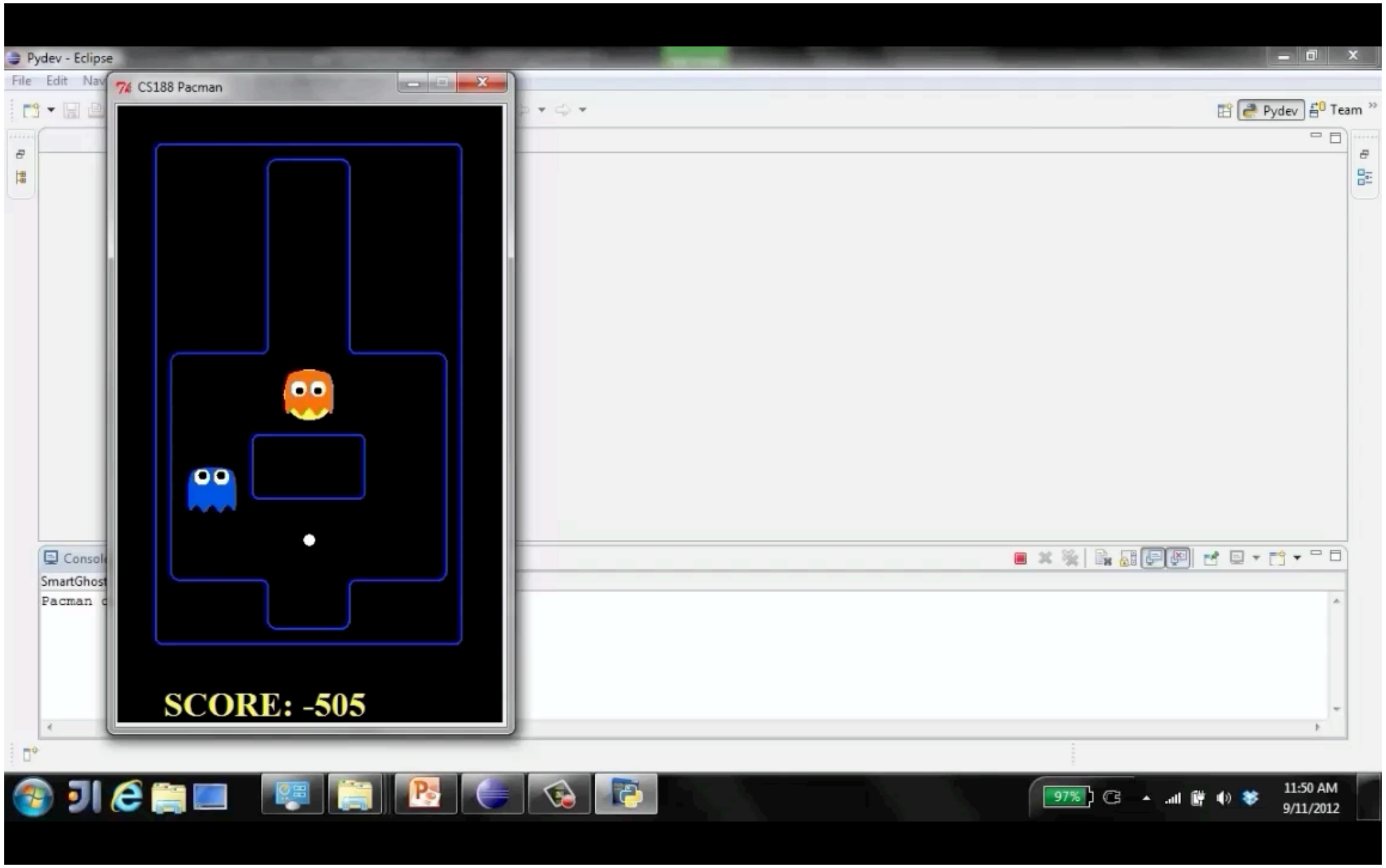
- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation



Video of Demo Limited Depth (2)



Video of Demo Limited Depth (10)



Synergies between Evaluation Function and Alpha-Beta?

- Alpha-Beta: amount of pruning depends on expansion ordering
  - Evaluation function can provide guidance to expand most promising nodes first (which later makes it more likely there is already a good alternative on the path to the root)
    - (somewhat similar to role of A* heuristic)

- Alpha-Beta: (similar for roles of min-max swapped)
  - Value at a min-node will only keep going down
  - Once value of min-node lower than better option for max along path to root, can prune
  - Hence: IF evaluation function provides upper-bound on value at min-node, and upper-bound already lower than better option for max along path to root
    THEN can prune