# *Assignment 1*

*Computer Science 235*

***Reading.*** Chapter 0 pages 1-28, Section 1.1 pages 31-47

**1)** Watch this video developed by a student who took Mt. Holyoke's version of Theory of Computation. Write down a short (3-4 sentences) reflection on what surprised you in this video (if anything), and what seems exciting about this coming semester.

**2)** Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

*a)* $\{1, 3, 5, 7, \ldots\}$

*b)* $\{w | w$ is a string of 0s and 1s and $w$ equals the reverse of $w\}$.

*c)* $\{n | n$ is an integer and $n = n + 1\}$

**3)** Write a formal description of the following sets.

*a)* The set containing all integers that are greater than 5.

*b)* The set containing the empty string.

*c)* The set containing nothing at all.

**4)** Let $A$ be the set $\{x, y, z\}$, and $B$ be the set $\{x, y\}$.

*a)* Is $A$ a subset of $B$?

*b)* Is $B$ a subset of $A$?

*c*) What is $A \cup B$?

*d*) What is $A \cap B$?

*e*) What is $A \times B$?

*f)* What is the power set of $B$?

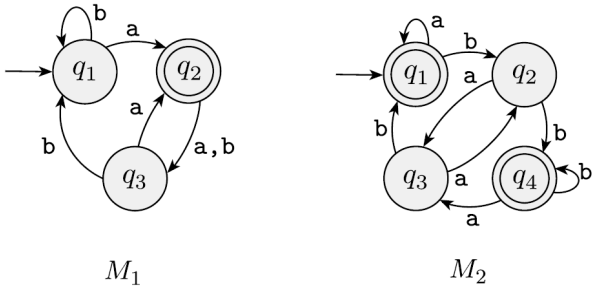
**5)** If $A$ has $a$ elements and $B$ has $b$ elements, how many elements are in $A \times B$? Explain your answer.

**6)** Let $L$ be the language over the alphabet {a,b} generated by the following recursive definition:
*Basis.* The empty string ε belongs to $L$.
*Recursive Step.* If $w$ is in $L$, then so is aa$w$b.
*Closure.* Any string $w$ in $L$ is either ε or else it can be obtained by the above recursive definition.

*a)* Give a formal symbolic description of the set of strings $L$ encompassed by the above definition. For example, you could state that $L = \{a^n b^3 \mid n = 1, 2, \ldots\}$, if this were an accurate description (which it is not).

*b)* Using the recursive definition of $L$ directly, prove by induction that if $w$ is in $L$ then a appears twice as many times as b in $w$.

**7)** The following are the state diagrams of two DFAs, $M_1$ and $M_2$.




$M_1$ $M_2$

Answer the following questions about these machines.

*a)* What sequence of states does $M_1$ go through on input aabb?

*b)* Does $M_1$ accept the string aabb?

*c)* Does $M_2$ accept the string ε?

**8)** Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is {0,1}.

*a)* $\{w \mid w$ begins with a 1 and ends with a 0$\}$.

*b)* $\{w \mid w$ contains at least three 1s$\}$.

c*)* $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.

d*)* $\{w \mid w$ doesn't contain the substring 110$\}$.

e) $\{w \mid$ every odd position of $w$ is a 1$\}$.

*f)* $\{w \mid w$ contains at least two 0s and at most one 1$\}$.

*g)* $\{\varepsilon, 0\}$.

*h)* The empty set.

*i)* All strings except the empty string.