Assignment 2

Computer Science 235

Reading. Section 1.2

1) Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. Assume the alphabet is {0, 1}.

a) The language {w | w contains the substring 0101} with five states.

b) The language {w | w contains an even number of 0s, or contains exactly two 1s} with six states.

c) The language 0*1*0*0 with three states.

d) The language {ε} with one state.

2) Assuming an alphabet of {0, 1}, consider two DFAs: one DFA recognizing the language {w | w begins with a 1 and ends with a 0} and another DFA recognizing the language {w | w contains at least three 1s}. Use the construction in the proof of Theorem 1.45 in Sipser to give the state diagram of an NFA recognizing the union of these two languages.

3) Assuming an alphabet of {0, 1}, consider two DFAs: one DFA recognizing the language {w | w contains at least three 1s} and another DFA recognizing the empty set. Use the construction in the proof of Theorem 1.47 in Sipser to give the state diagram of an NFA recognizing the concatenation of these two languages.

4) Assuming an alphabet of {0, 1}, consider a DFA recognizing the language {w | w contains at least three 1s}. Use the construction in the proof of Theorem 1.49 in Sipser to give the state diagram of an NFA recognizing the star of this language.

5) Show by giving an example that if N is an NFA that recognizes language A, swapping the accept and nonaccept states in N doesn't necessarily yield a new NFA that recognizes the complement of A. Is the class of languages recognized by NFAs closed under complementation? Explain your answer.

6) For any string $w = w_1 w_2 \cdots w_n$, the ***reverse*** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in A\}$. Show that if A is regular, so is $A^{\mathcal{R}}$.

7) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with $B^{\mathcal{R}}$ is easier. You may assume the result claimed in problem **(6)** above).