Assignment 3  
Computer Science 235

Reading. Section 1.3

1) Use the construction given in Theorem 1.39 in Sipser to convert the following nondeterministic finite automaton to an equivalent deterministic finite automaton.

2) Give regular expressions generating the following languages.
   a) \( \{w \mid w \text{ begins with a 1 and ends with a 0}\} \)
   b) \( \{w \mid w \text{ contains at least three 1s}\} \)
   c) \( \{w \mid w \text{ contains the substring 0101}\} \)
   d) \( \{w \mid w \text{ has length at least 3 and its third symbol is a 0}\} \)
   e) \( \{w \mid w \text{ starts with a 0 and has odd length, or starts with a 1 and has even length}\} \)

3) Use the procedure described in Lemma 1.55 in Sipser to convert the following regular expressions to nondeterministic finite automata.
   a) \(((00)^*(11)) \cup 01)^*\)
   b) \(\emptyset^*\)

4) For each of the following languages, give two strings that are members and two strings that are not members - a total of four strings for each part. Assume the alphabet \( \Sigma = \{a, b\} \).
   a) \(a(ba)^*b\)
   b) \(a^* \cup b^*\)
   c) \(\Sigma^a\Sigma^*b\Sigma^*a\Sigma^*\)
   d) \((\epsilon \cup a)b\)

5) Use the procedure described in Lemma 1.60 in Sipser to convert the following finite automaton to a regular expression.
6) A **finite state transducer** (FST) is a type of deterministic finite automaton whose output is a string and not just accept or reject. The following is a state diagram of a finite transducer $T$.

Each transition of an FST is labeled with two symbols, one designating the input symbol for that transition and the other designating the output symbol. The two symbols are written with a slash, $/$, separating them. In $T$, the transition from $q_1$ to $q_2$ has input symbol $a$ and output symbol $1$. When an FST computes on an input string $w$, it takes the input symbols $w_1 \cdots w_n$ one by one and, starting at the start state, follows the transitions by matching the input labels with the sequence of symbols $w_1 \cdots w_n = w$. Every time it goes along a transition, it outputs the corresponding output symbol. For example, on input `abbb`, $T$ enters the sequence of states $q_1, q_2, q_1, q_3, q_2$ and produces output `1011`. Give (a) the sequence of states entered and (b) the output produced by $T$ for the input `bbab`.

7) Give the formal definition of the FST model (see previous problem for definition of FST), following the pattern in Definition 1.5 in Sipser. Assume that an FST has an input alphabet $\Sigma$ and an output alphabet $\Gamma$ but not a set of accept states. Include a formal definition of the computation of an FST, following the pattern on page 40 in Sipser. (Hint: An FST is a 5-tuple. Its transition function is of the form $\delta: Q \times \Sigma \rightarrow Q \times \Gamma$.)

8) Recall that string $x$ is a **prefix** of string $y$ if a string $z$ exists where $xz = y$, and that $x$ is a **proper prefix** of $y$ if in addition $x \neq y$. Consider the following operation on a language $A$:

$$
\text{NOEXTEND}(A) = \{ w \in A \mid w \text{ is not the proper prefix of any string in } A \}
$$
Show that the class of regular languages is closed under the \textit{NOEXTEND} operation.

9) For languages $A$ and $B$, let the \textit{perfect shuffle} of $A$ and $B$ be the language

$$\{w \mid w = a_1b_1\cdots a_kb_k, \text{ where } a_1\cdots a_k \in A \text{ and } b_1\cdots b_k \in B, \text{ with each } a_i, b_i \in \Sigma\}.$$ 

Show that the class of regular languages is closed under perfect shuffle.

10) \textbf{PROJECT.} This is a project that you will engage with throughout the rest of the semester. The project asks you to interact with papers written in (or adjacent to) the field of Theory of Computation.

We have chosen 5 papers for us to work with as a class this semester. Each paper will have several students assigned to it, so you'll be able to talk with your peers about the information in the paper, your questions on it, etc. Once your paper is assigned to you, you will employ different techniques in order to read this paper (from skimming, to careful reading with looking up unknown words, or engaging in a peer discussion). At the end of the semester you will submit a 1-2 page reflection on the process of reading the research paper that you were assigned, what you have found challenging, what you have found interesting, and what new questions it has made you ask yourself.

This is an opportunity for you to try something really challenging in a supportive environment. This is a different type of reading, and for some of you it may be a completely new type of reading.

\textbf{Part 1 (of 3).} Read the abstract of each of the five papers below (in the case of the fifth paper, "E) Turing", read the first section).

- A) Game Theory
- B) Linguistics
- C) AI Epistemology
- D) Nature of ToC (Theory of Computation)
- E) Turing

Now fill out this \textbf{Google form} indicating the top three papers, in ranked order, that you are most interested in. Over the next week or two, the instructor will assign you one of these papers.

\textbf{Grading}

This project is worth 5% of your final grade. You will be graded on the final reflection that you submit as part of Assignment 9.