Reading. Section 1.4

1) The pumping lemma says that every regular language has a pumping length $p$, such that every string in the language can be pumped if it has length $p$ or more. If $p$ is a pumping length for language $A$, so is any length $p' \geq p$. The minimum pumping length for $A$ is the smallest $p$ that is a pumping length for $A$. For example, if $A = 01^*$, the minimum pumping length is 2. The reason is that the string $s = 0$ is in $A$ and has length 1 yet $s$ cannot be pumped; but any string in $A$ of length 2 or more contains a 1 and hence can be pumped by dividing it so that $x = 0$, $y = 1$, and $z$ is the rest. For each of the following languages, give the minimum pumping length.

   a) $0001^*$
   b) $001 \cup 0^*1^*$
   c) $(01)^*$
   d) $001$

2) Prove that the following languages are not regular.
   a) $\{ 0^n1^m0^n \mid m,n \geq 0 \}$
   b) The complement of $\{ 0^n1^n \mid n \geq 0 \}$
   c) $\{ w \mid w \in \{0,1\}^* \text{ is not a palindrome} \}$
   d) $\{ wtw \mid w,t \in \{0,1\}^+ \}$

3) a) Let $B = \{ 1^kg \mid g \in \{0,1\}^* \text{ and } g \text{ contains at least } k \text{ 1s, for } k \geq 1 \}$. Show that $B$ is a regular language.
   b) Let $C = \{ 1^kg \mid g \in \{0,1\}^* \text{ and } g \text{ contains at most } k \text{ 1s, for } k \geq 1 \}$. Show that $C$ is not a regular language.

4) Consider the language $F = \{ d^ib^jc^k \mid i,j,k \geq 0 \text{ and if } i = 1 \text{ then } j = k \}$.
   a) Show that $F$ is not regular.
   b) Show that $F$ acts like a regular language in the pumping lemma. In other words, give a pumping length $p$ and demonstrate that $F$ satisfies the three conditions of the pumping length for this variable $p$.
   c) Explain why parts (a) and (b) do not contradict the pumping lemma.