Assignment 4

Computer Science 235

Reading. Section 1.4

1) The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length for language A, so is any length $p' \geq p$. The ***minimum pumping length*** for A is the smallest p that is a pumping length for A. For example, if A = `01*`, the minimum pumping length is 2. The reason is that the string s = `0` is in A and has length 1 yet s cannot be pumped; but any string in A of length 2 or more contains a `1` and hence can be pumped by dividing it so that x = `0`, y = `1`, and z is the rest. For each of the following languages, give the minimum pumping length.

a) `0001*`
b) `001 ∪ 0*1*`
c) `(01)*`
d) `001`

2) Prove that the following languages are not regular.

a) $\{ 0^n 1^m 0^n \mid m,n \geq 0 \}$
b) The complement of $\{ 0^n 1^n \mid n \geq 0 \}$
c) $\{ w \mid w \in \{0,1\}^*$ is not a palindrome $\}$
d) $\{ wtw \mid w,t \in \{0,1\}^+ \}$

3) *a)* Let $B = \{ 1^k g \mid g \in \{0,1\}^*$ and g contains at least k 1s, for $k \geq 1 \}$. Show that B is a regular language.
b) Let $C = \{ 1^k g \mid g \in \{0,1\}^*$ and g contains at most k 1s, for $k \geq 1 \}$. Show that C is not a regular language.

4) Consider the language $F = \{ a^i b^j c^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k \}$.

a) Show that F is not regular.
b Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping length for this variable p.
c) Explain why parts (*a*) and (*b*) do not contradict the pumping lemma.