Reading. Sections 4.1 and 4.2

1) Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

2) Let $A_{\epsilon_{\text{CFG}}} = \{<G> \mid G \text{ is a CFG that generates } \epsilon\}$. Show that $A_{\epsilon_{\text{CFG}}}$ is decidable.

3) Let $\Psi$ be the set of all infinite sequences over $\{0, 1\}$. Show that $\Psi$ is uncountable using a proof by diagonalization.

4) A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

5) Let $A = \{<R, S> \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that $A$ is decidable.

6) Let $R$ be a regular expression and let $G$ be a CFG. Show that the problem of determining whether there exists a string that is generated by both $R$ and $G$ is decidable (note, we are not checking if a specific string $w$ is generated by both $R$ and $G$ but rather if there exists any string that is generated by both $R$ and $G$).