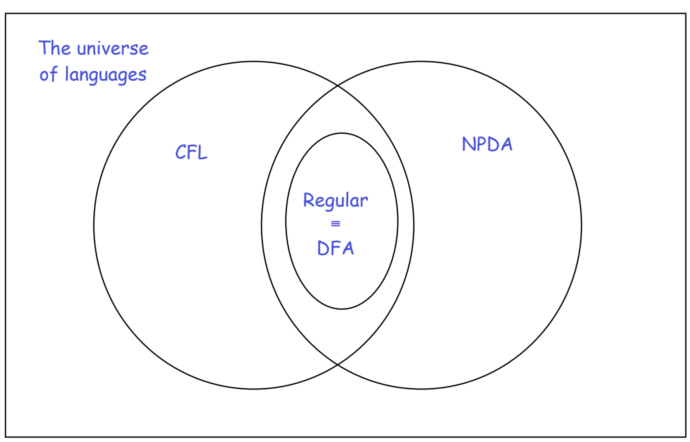
Chomsky hierarchy of languages

Equivalence of CFL and NPDA

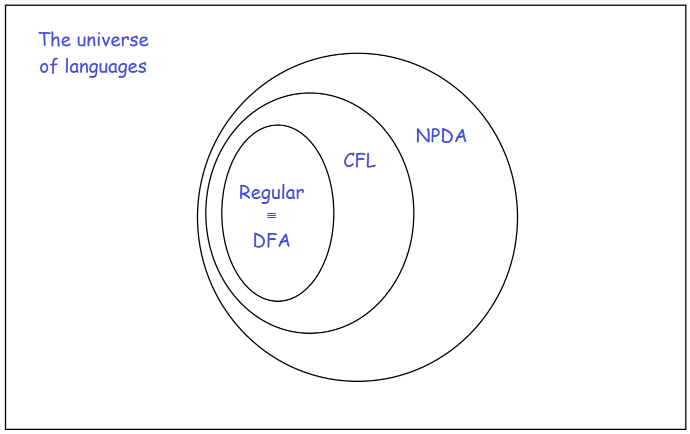
CS235 Languages and Automata

Department of Computer Science
Wellesley College

More worries



A partial answer



Constructing NPDAs from CFGs

Given a CFG $G = (N, \Sigma, P, S)$ in Greibach normal form, define

$M = (\{q\}, \Sigma, N, \Delta, q, S, \emptyset)$,

where

- q is the only state;
- Σ, terminals of G, is the input alphabet;
- N, nonterminals of G, is stack alphabet;
- q is the start state;
- S; start symbol of G, is the initial stack symbol;
- $\emptyset$, the set of final states is empty.
- Δ, transition relation is defined as follows; For each production

$$A \rightarrow cB_1B_2 \dots B_k$$

in P, Δ contains the transition

$$((q, c, A), (q, B_1B_2 \dots B_k)).$$

Nonnull balanced strings of parentheses

Recall we tried this construction on our old friend:

(i)	*S* → [*BS*
(ii)	*S* → [*B*
(iii)	*S* → [*SB*
(iv)	*S* → [*SBS*
(v)	*B* → [

Nonnull balanced strings of parentheses

The result:

(i)	*S* → [*BS*	((*q*, [, *S*), (*q*, *BS*))
(ii)	*S* → [*B*	((*q*, [, *S*), (*q*, *B*))
(iii)	*S* → [*SB*	((*q*, [, *S*), (*q*, *SB*))
(iv)	*S* → [*SBS*	((*q*, [, *S*), (*q*, *SBS*))
(v)	*B* → [	((*q*, [, *B*), (*q*, ε))

Parsing [[[ ]] [ ]]

Rule applied	Sentential forms in leftmost derivation	Configurations of M in accepting computation
	S	$(q,$ [[[ ]] [ ]], $S)$
(iii)	[SB	$(q,$ [[ ]] [ ]], $SB)$
(iv)	[[$SBSB$	$(q,$ [ ]] [ ]], $SBSB)$
(ii)	[[[$BBSB$	$(q,$]] [ ]], $BBSB)$
(v)	[[[ ] BSB	$(q,$] [ ]], $BSB)$
(v)	[[[ ]] SB	$(q,$ [ ]], $SB)$
(ii)	[[[ ]] [BB	$(q,$]], $BB)$
(v)	[[[ ]] [ ] B	$(q,$], $B)$
(v)	[[[ ]] [ ]]	$(q, \varepsilon, \varepsilon)$

Derivations and configurations

Lemma 24.1. For any $z, y \in \Sigma^*$, $\alpha \in N^*$, and $A \in N$,

$$A \overset{n}{\underset{G}{\Rightarrow}} z\alpha$$

via a leftmost derivation if and only if

$$(q, zy, A) \overset{n}{\underset{M}{\vdash}} (q, y, \alpha).$$

How this helps

Theorem 24.2. *L(M)* = *L(G)*.

Proof.

$x \in L(G)$	$\Leftrightarrow \; S \overset{n}{\underset{G}{\Rightarrow}} x$ by a leftmost derivation	definition of *L(G)*
	$\Leftrightarrow \; (q, x, S) \overset{n}{\underset{M}{\vdash}} (q, \varepsilon, \varepsilon)$	Lemma 24.1
	$\Leftrightarrow \; x \in L(M)$	definition of *L(M)*.

Roll up your sleeves

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Proof (by induction on *n*).