Predictive Parsing

Monday, November 19, 2007
and Monday November 26, 2007
Reading: Appel 3.2

CS235 Languages and Automata

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Overview

For some grammars, reading the first token (or first few tokens) of input is sufficient for determining which production to apply.

For such grammars, we can write a so-called **predictive parser**.

Today, we will see how to determine if a grammar is amendable to predictive parsing and how to build a predictive parser by hand.

We will also learn some techniques for handling grammars for which predictive parsers do not exist.

Running Example: SLiP--

As our running example, we will use SLiP--, a subset of Appel's straight-line programming language (SLiP).

The **abstract syntax** of SLiP-- is described by these SML datatypes:

```
datatype pgm = Pgm of Stm
and  stm = Assign of string * exp
         |   Print of exp
         |   Seq of stm list
and exp = Id of string
         |  Int of int
         |  BinApp of exp * binop * exp
and binop = Add | Sub | Mul | Div
```

We will explore several versions of **concrete syntax** for SLiP--

Our First Concrete Syntax for SLiP--

Productions for Concrete Grammar

P → S EOF

S → ID(str) := E | print E | begin SL end

SL → % | S ; SL

E → ID(str) | INT(int) | (E B E)

B → + | - | * | /

SML Data Types for Abstract Grammar

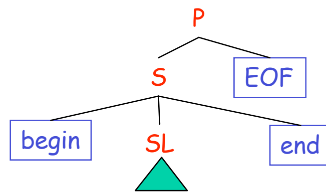
```
datatype pgm = Pgm of Stm
and stm = Assign of string * exp
        |   Print of exp
        |   Seq of stm list
and exp = Id of string
         |  Int of int
         |  BinApp of exp * binop * exp
and binop = Add | Sub | Mul | Div
```

An Example SLiP-- Program

Chars: begin x := (3+4); print ((x-1)*(x+2)); end

Tokens: begin ID("x") := (INT(3) + INT(4)) ;
print ((ID("x") - INT(1)) * (ID("x") +
INT(2))) ; end EOF

Parse Tree:
(see full tree
on next slide)

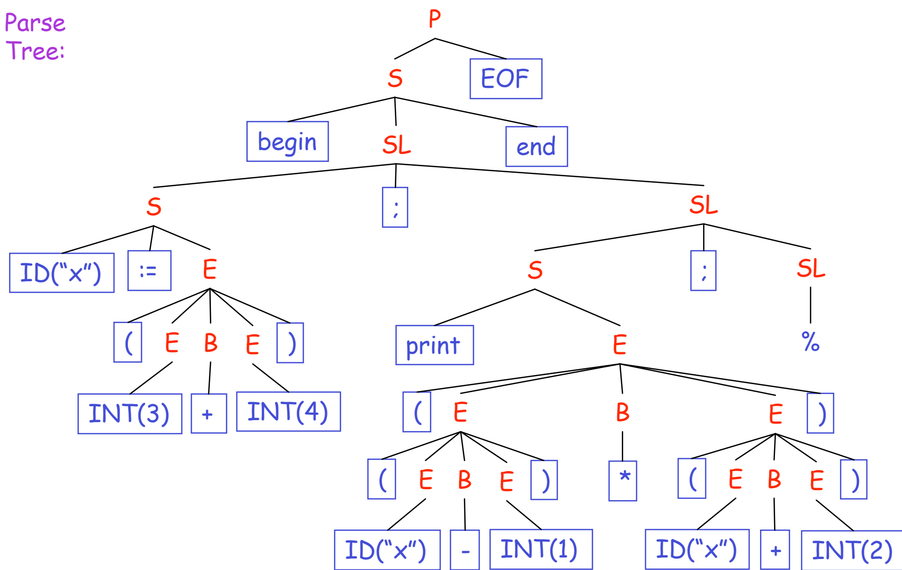


Abstract Syntax Tree (AST):

```
Pgm(Seq [Assign(BinApp(Int(3),Add,Int(4))),
         Print(BinApp(BinApp(Id("x"),Sub,Int(1)),
                      Mul,
                      BinApp(Id("x"),Add,Int(2))))])
```


An Example SLiP-- Program

Parse Tree:



Predictive Parsing For SLiP--

P → S EOF

S → ID(str) := E | print E | begin SL end

SL → % | S ; SL

E → ID(str) | INT(int) | (E B E)

B → + | - | * | /

Observe that:

- expressions (E) must begin with ID(str), INT(int), or (
- statements (S) must begin with ID(str), print, or begin
- statement lists (SL) must begin with a statement (S) and so must begin with ID(str), prin, or begin . They must end with end (a token that is not part of the SL tree but one immediately following it).
- programs (P) must begin with a statement (S) and so must begin with ID(str) , print , or begin

Predictive Parsing Table for SLiP--

Can summarize observations on previous slide with a **predictive parsing table** of variables x tokens in which at most one production is valid per entry.

Empty slots in the table indicate parsing errors.

	ID(s)	INT(i)	(	OP(b)	print	begin	end
P	P → S EOF				P → S EOF	P → S EOF	
S	S → ID(str) := E				S → print E	S → begin SL end	
SL	SL → S ; SL				SL → S ; SL	SL → S ; SL	SL → %
E	E → ID(str)	E → INT(num)	E → (E B E)				
B				B → OP(b)			

Recursive Descent Parsing

From a predictive parsing table, it is possible to construct a **recursive descent parser** that parses tokens according to productions in the table.

Such a parser can eat (consume) or "peek" at the next token.

See ParserParens.sml for a recursive descent parser for SLiP-- written in SML.

NULLABLE, FIRST, and FOLLOW

Parsing tables are constructed using the following notions:

Let t range over terminals, V and W range over variables, α range over terminals $\cup$ variables, and γ range over sequences of terminals $\cup$ variables.

- **NULLABLE(γ)** is true iff γ can derive the empty string (%)
- **FIRST(γ)** is the set of terminals that can begin strings derived from γ.
- **FOLLOW(V)** is the set of terminals that can immediately follow V in some derivation.

Computing NULLABLE For Variables

A variable V is NULLABLE iff

1. There is a production $V \to \%$

OR

2. There is a production $V \to V_1 \ldots V_n$
   and each of $V_1, \ldots, V_n$ is NULLABLE

(Case 1 is really a special case of 2 with n = 0.)

In general, it is necessary to compute an **iterative fixed point** to determine nullability of a variable.

Example (from Appel 3.2)

$X \to a \mid Y$

$Y \to \% \mid c$

$Z \to d \mid X\ Y\ Z$

Computing FIRST

FIRST(t) = t

FIRST(V) = $\cup$ {FIRST(γ) | $V \to \gamma$ is a production for V}

FIRST($\alpha_0 \ldots \alpha_i \ldots \alpha_n$) = $\cup$ {FIRST(α_i) | $\alpha_0, \ldots, \alpha_{i-1}$ are all nullable}

Again, this is determined by an **iterative fixed point** computation

$X \to a \mid Y$

$Y \to \% \mid c$

$Z \to d \mid X\ Y\ Z$

Computing FOLLOW

FOLLOW(V) =
$\cup$\{FIRST(α_j) | W → $\alpha_0 \ldots \alpha_{i-1}$ V $\alpha_{i+1} \ldots \alpha_j \ldots \alpha_n$
is a production in the grammar
and $\alpha_{i+1}, \ldots, \alpha_{j-1}$ are all nullable\}
$\cup$ $\cup$\{FOLLOW(W) | W → $\alpha_0 \ldots \alpha_{i-1}$ V$\alpha_{i+1} \ldots \alpha_n$
is a production in the grammar
and $\alpha_{i+1}, \ldots, \alpha_n$ are all nullable\}

Again, this is determined by an **iterative fixed point** computation

X → a | Y
Y → % | c
Z → d | X Y Z

Constructing Predictive Parsing Tables

To construct a predictive parsing table,
do the following for each production V → γ:

- For each t in FIRST(γ), enter V → γ in row V, column t.
- If NULLABLE(γ), for each t in FOLLOW(V), enter V → γ in row V, column t

P → S EOF
S → ID(str) := E | print E | begin SL end
SL → % | S ; SL
E → ID(str) | INT(int) | (E B E)
B → + | - | * | /

More Practice

S' → S EOF
S → T | 0S1
T → % | 10T

	NULLABLE	FIRST	FOLLOW
S'			
S			
T			

	0	1	EOF
S'			
S			
T			

Parsing not predictive since some table slots now have multiple entries!

Adding Extra Lookahead

S' → S EOF
S → T | 0S1
T → % | 10T

Sometimes predictivity can be re-established by adding extra **lookahead**:

	0	10	11	1 EOF	EOF
S'					
S					
T					

LL(k) Grammars

An LL(k) grammar is one that has a predictive parsing table with k symbols of lookahead.

- The SLiP-- grammar is LL(1).
- The S'/S/T grammar is LL(2) but not LL(1).

In LL,

- the first L means the tokens are consumed left-to-right.
- the second L means that the parse tree is constructed in the manner of a leftmost derivation.

SLiP-- Expressions with Prefix Syntax

Suppose we change Slip-- expressions to use prefix syntax:

E → ID(str) | INT(int) | B E E

Parsing is still predictive:

	ID(s)	INT(i)	OP(b)	print	begin	end
E	E → ID(str)	E → INT(num)	E → B E E			
B			B → OP(b)			

Postfix Syntax for Expressions

Suppose we change Slip-- expressions to use postfix syntax:

E → ID(str) | INT(int) | E E B

Parsing is no longer predictive since some table slots now have multiple entries:

	ID(s)	INT(i)	OP(b)	print	begin	end
E	E → ID(str) E → E E B	E → INT(num) E → E E B				
B			B → OP(b)			

Postfix expressions are fundamentally not predictive (not LL(k) for any k) there's nothing we can do to parse them predictively.

But we'll soon see we *can* parse them with a shift/reduce parser.

Infix Syntax for Expressions

Suppose we change Slip-- expressions to use infix syntax without required parens (but with optional ones)

E → ID(str) | INT(int) | E B E | (E)

Parsing is no longer predictive:

	ID(s)	INT(i)	OP(b)	(	print	begin	end
E	E → ID(str) E → E B E	E → INT(num) E → E B E		E → (E)			
B			B → OP(b)				

This is not surprising: this grammar is ambiguous, and *no* ambiguous grammar can be uniquely parsed with *any* deterministic parsing algorithm.

Removing Ambiguity May not Help

Supppose we use an unambiguous infix grammar for arithmetic:

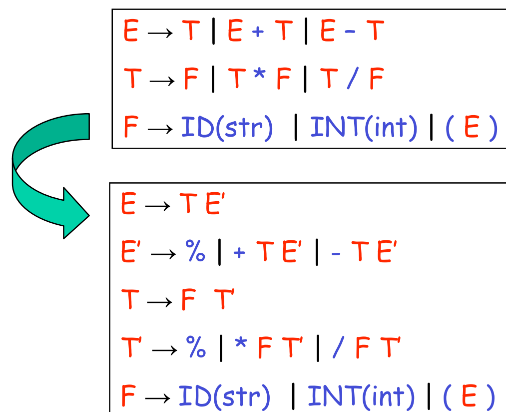
E → T | E + T | E - T	*Expressions*
T → F | T * F | T / F	*Terms*
F → ID(str) | INT(int) | (E)	*Factors*

Parsing is *still* not predictive due to **left recursion** in E and T:

	ID(s)	INT(i)	OP(b)	(	print	begin	end
E	E → T E → E + T E → E - T	E → T E → E + T E → E - T		E → T E → E + T E → E - T			
T	T → F T → T * F T → T / F	T → F T → T * F T → T / F		T → F T → T * F T → T / F			
F	F → ID(str)	F → INT(num)		F → (E)			

Left Recursion Removal

Sometimes we can transform a grammar to remove left recursion (parse trees are transformed correspondingly).



See Appel 3.2 for a general description of this transformation.
You will see an example of this in PS5.

The Transformed Grammar is Predictive!

E → T E'
E' → % | + T E' | - T E'
T → F T'
T' → % | * F T' | / F T'
F → ID(str) | INT(int) | (E)

	ID(s)	INT(i)	+	*	(	)	;	EOF
E	E → T E'	E → T E'			E → T E'			
E'			E' → + T E'			E' → %	E' → %	E' → %
T	T → F T'	T → F T'			T → F T'			
T'			T' → %	T' → * F T'		T' → %	T' → %	T' → %
F	F → ID(str)	F → INT(num)			F → (E)			

Transforming Parse Trees

The parse tree from the transformed grammar can be transformed back to the untransformed grammar.
E.g. 2 * 3 + 4