

Problem Set 5

Due: 11:59 pm on Tuesday, December 4

This original version of this problem set is the final one. Problem 6 has been moved to the (Optional) Problem Set 6.

Required Reading: Stoughton, Sections 4.1, 4.3, 4.6–4.10;

Strongly Suggested Reading: Sipser, Chapter 2

Submission:

You should turn in a hardcopy submission packet by slipping it under Lyn's office door by 11:59pm on the due date. This packet should include your written solutions for Problems 1 through 5;

There is no softcopy submission for this problem set.

Problem 1 [60]: CFGs and NPDAs

For each of the following languages, do *both* of the following:

1. Construct a context-free grammar (CFG) that generates the language.
2. Construct a nondeterministic pushdown automaton (NPDA) that accepts the language.

a [10] $L_1 = \{a^i b^j \mid i, j \in \text{Nat and } i < j\}$

b [10] $L_2 = \{w \mid w \in \{a, b\}^* \text{ and } w = w^R\}$

c [15] $L_3 = \{w \mid w \in \{a, b\}^* \text{ and } w \text{ has twice as many as as bs}\}$

d [25] $L_4 = \{a, b\}^* - \{a^n b^n \mid n \in \text{Nat}\}$. (Hint: what are all the possible forms of strings in L_4 ?)

Problem 2 [25]: Am-bug-uity

Consider the language

$$L_{\#a=\#b} = \{w \mid w \in \{a, b\}^* \text{ and } w \text{ contains an equal number of as and bs}\}$$

In the lecture on Ambiguous Grammars, slide 25-3 claims that the following context free grammar CFG_{buggy} is an unambiguous grammar for this language:

$$\begin{aligned} S &\rightarrow \epsilon \mid CS \\ C &\rightarrow aSb \mid bSa \end{aligned}$$

a [4] Show that this claim is *incorrect* by giving two parse trees for the string **abab** using CFG_{buggy} .

b [8] Construct a context free grammar $CFG_{correct}$ that is an unambiguous grammar for $L_{\#a=\#b}$. Hint: you need only three variables.

c [4] Briefly explain why $CFG_{correct}$ generates $L_{\#a=\#b}$

d [4] Briefly explain why $CFG_{correct}$ is unambiguous.

e [5] Using $CFG_{correct}$, give the unique parse tree for **aaababbbbbbbaabaaab**.

Problem 3 [15]: Chomsky Normal Form

Convert the following grammar to Chomsky Normal Form:

$$X \rightarrow a \mid Y \mid XYZ$$

$$Y \rightarrow \% \mid bZ \mid Yc$$

$$Z \rightarrow d \mid Ye$$

Problem 4 [40]: Language Reversals and Left-Linear Grammars

a [3] Suppose that the context free grammar CFG generates language L_{CFG} . Describe how to convert CFG to a context free grammar CFG' that generates L_{CFG}^R .

b [3] Use your conversion algorithm from part **a** to convert the following CFG to one that generates the reverse language:

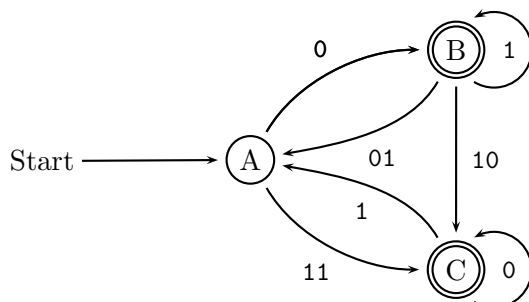
$$A \rightarrow A10 \mid B1 \mid C0$$

$$B \rightarrow \% \mid A11 \mid B01 \mid C101$$

$$C \rightarrow \% \mid B010 \mid C00$$

c [5] Suppose that the finite automaton FA accepts language L_{FA} . Describe how to convert FA to a finite automaton FA' that accepts L_{FA}^R .

d [5] Use your conversion algorithm from part **c** to convert the following FA to one that accepts the reverse language.



e [10] Recall that

- A CFG is **right-linear** if every production with variables in the right-hand side (RHS) has exactly one variable in the RHS, and it comes last.
- A CFG is **left-linear** if every production with variables in the RHS has exactly one variable in the RHS, and it comes first.
- A language is regular iff it is generated by a right-linear grammar.

Using the above definitions and facts and the results of parts **a** and **c**, prove that a language is regular iff it is generated by a left-linear grammar.

f [7] Construct a finite automaton that accepts the language generated by the left-linear grammar in part **b**.

g [7] Construct a left-linear grammar that generates the language accepted by the finite automaton in part **d**.

Problem 5 [45]: Non-Context-Free Languages

Prove that each of the following languages is not context-free. You may use the pumping lemma for context-free languages and the fact that context-free languages are closed under intersection with a regular language, under concatenation, and under union. You may also use any facts about particular languages from lectures, readings, or other parts of this assignment.

a [5] $L_5 = \{w \mid w \in \{a, b, c\}^* \text{ and } w \text{ has an equal number of } a\text{s, } b\text{s, and } c\text{s}\}$

b [8] $L_6 = \{w \mid w \in \{a\}^* \text{ and } |w| = n^2 \text{ for some } n \in \text{Nat.}\}$

c [12] $L_7 = \{w \mid w \in \{a, b, c, d\}^* \text{ and } w \text{ has an equal number of } a\text{s and } b\text{s}$
 $\text{and } w \text{ has an equal number of } c\text{s and } d\text{s}\}$

d [15] $L_8 = \{a^n b^{2n} a^n \mid n \in \text{Nat}\}$

e [5] $L_9 = \{w \mid w \in \{a, b\}^* \text{ and } w = w^R \text{ and } w \text{ has an equal number of } a\text{s and } b\text{s}\}$