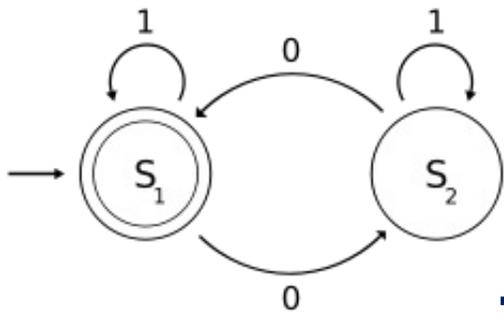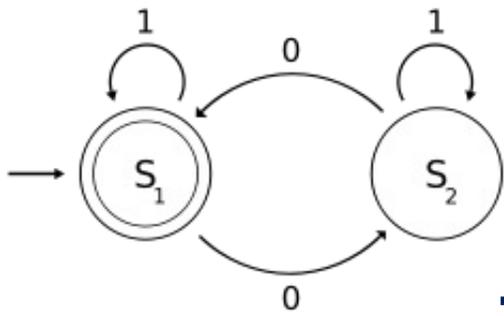# Context-Free and Pushdown Recognition
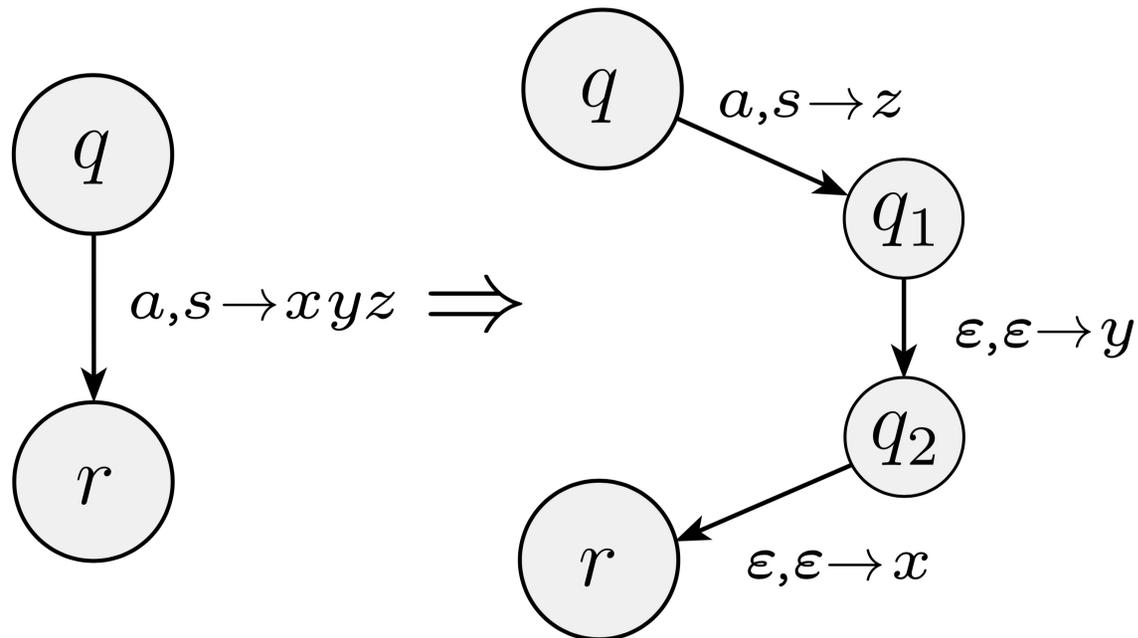
PDA Example

Show the state diagram for a PDA that recognizes $\{\, 0^n 1^n \mid n \geq 0 \,\}$.

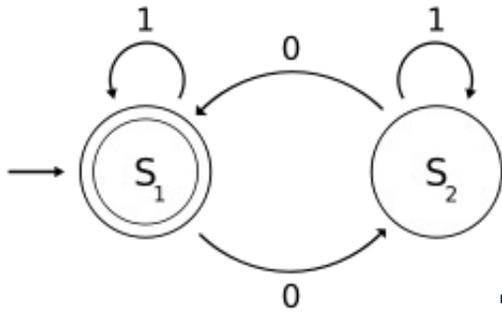# Writing Strings to Stack

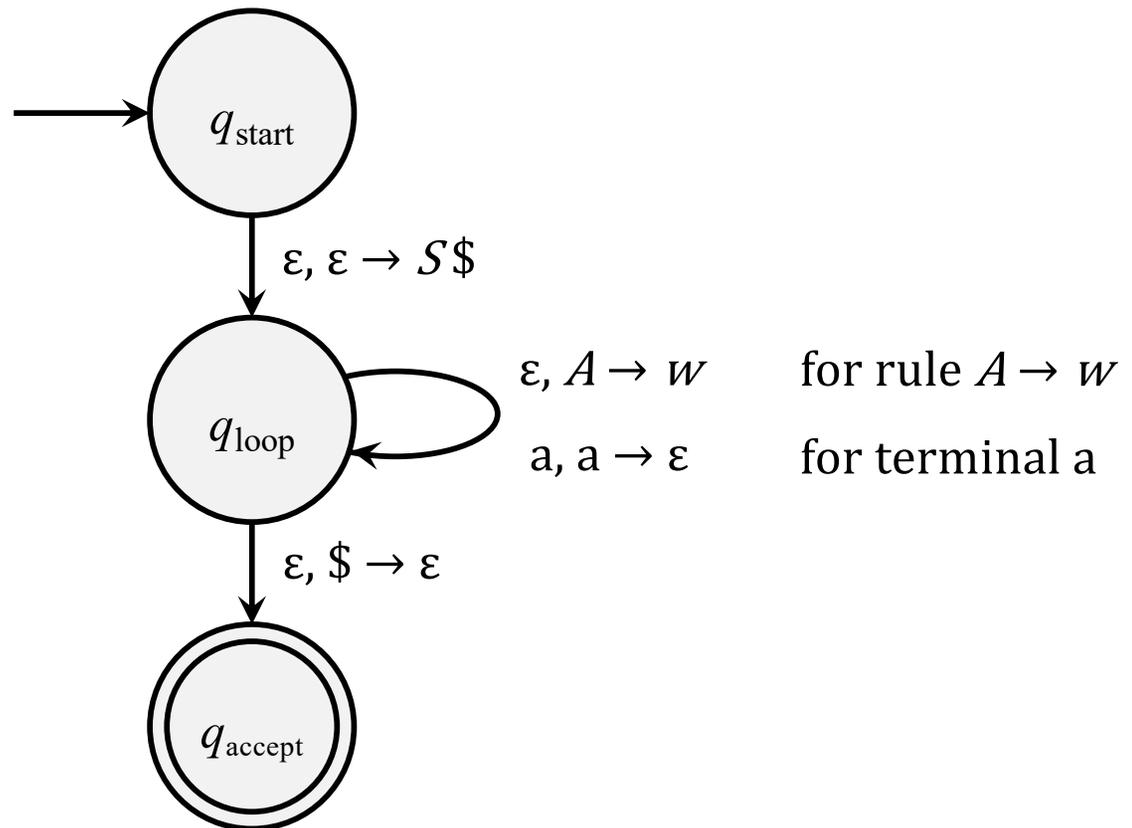Suppose we want to write strings (multiple characters) to a PDA stack...

q

$a,s \rightarrow xyz$

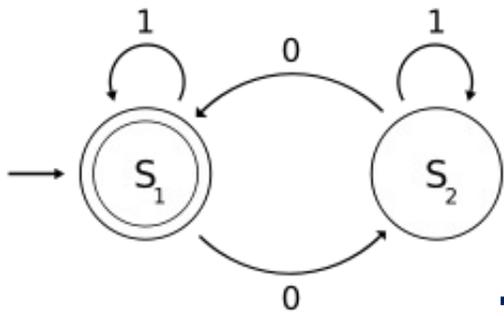r

$\Longrightarrow$

q $a,s \rightarrow z$

q_1

$\varepsilon,\varepsilon \rightarrow y$

q_2

r $\varepsilon,\varepsilon \rightarrow x$

Recognizing Context-Free Languages

Lemma. If a language is context-free, then some pushdown automaton recognizes it.

Proof.

q_{start}

$\varepsilon, \varepsilon \rightarrow S\$$

q_{loop}

$\varepsilon, A \rightarrow w$ for rule $A \rightarrow w$

a, a $\rightarrow \varepsilon$ for terminal a

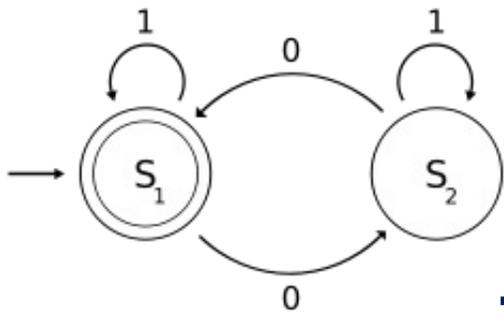$\varepsilon, \$ \rightarrow \varepsilon$

q_{accept}

For Example

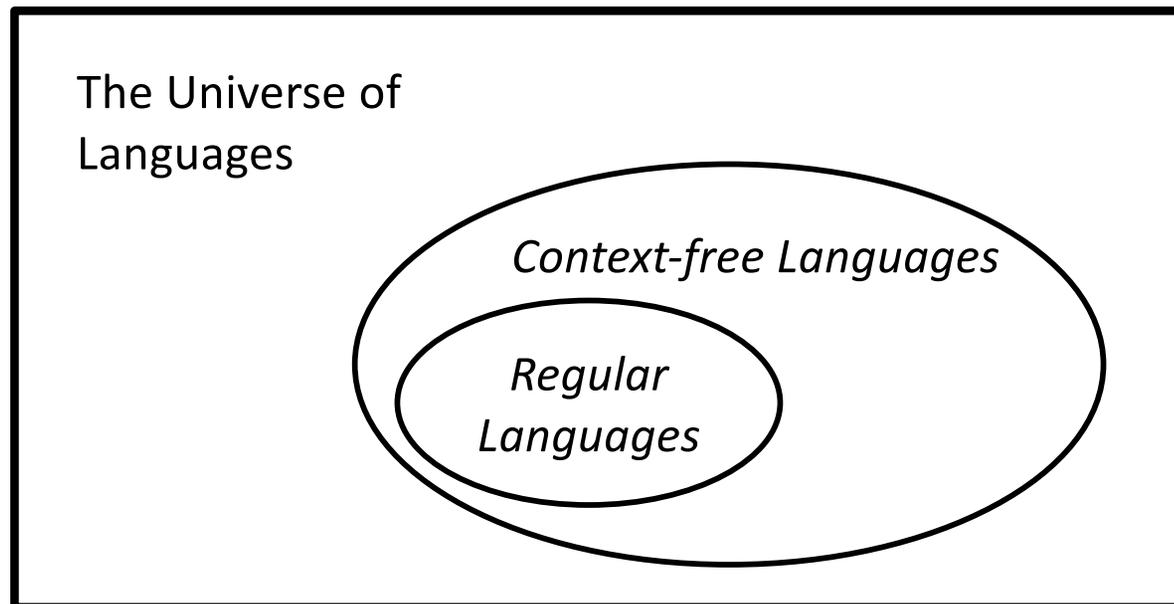We apply this construction to $G = (V, \Sigma, R, S)$, where
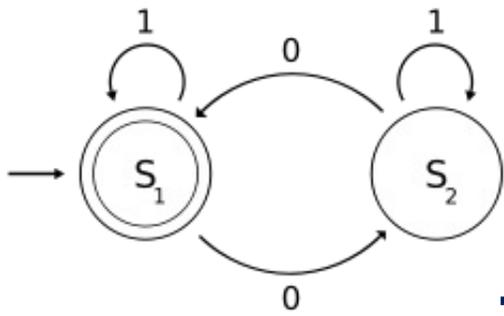
$V = \{S\}$,

$\Sigma = \{[,]\}$,

$R = \{ S \rightarrow \varepsilon \mid SS \mid [S] \}$.

Chomsky Hierarchy of Languages (Partial)

Corollary.   Every regular language is context-free.

The Universe of Languages

Context-free Languages

Regular Languages

Exercise

Convert each of the two grammars below to an equivalent PDA using the procedure just described.

$$S \rightarrow \mathtt{a}T\mathtt{b} \mid \mathtt{b}$$
$$T \rightarrow T\mathtt{a} \mid \varepsilon$$

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T \times F \mid F$$
$$F \rightarrow (E) \mid \mathtt{a}$$